Supporting California's Standards

Mathematics Bookmarks

Standards Reference to Support Planning and Instruction

4th Grade

Tulare County Office of Education

Tim A. Hire, County Superintendent of Schools

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Grade-Level Introduction

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

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(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

FLUENCY
In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., "fluently" multiply multi-digit whole numbers using the standard algorithm (5.NBT.5▲). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary. The word "fluent" is used in the standards to mean "reasonably fast and accurate" and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major3 [m] clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. The ▲ symbol will indicate standards in a Major Cluster in the narrative.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade Supporting [s] clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

California *Mathematics Framework*, adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

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Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Mathematical Practices

1. **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

Students:	Teachers:
• Analyze and explain the meaning of the problem • Actively engage in problem solving (Develop, carry out, and refine a plan) • Show patience and positive attitudes • Ask if their answers make sense • Check their answers with a different method	• Pose rich problems and/or ask open ended questions • Provide wait-time for processing/finding solutions • Circulate to pose probing questions and monitor student progress • Provide opportunities and time for cooperative problem solving and reciprocal teaching

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2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.

Students:	Teachers:
• Represent a problem with symbols • Explain their thinking • Use numbers flexibly by applying properties of operations and place value • Examine the reasonableness of their answers/calculations	• Ask students to explain their thinking regardless of accuracy • Highlight flexible use of numbers • Facilitate discussion through guided questions and representations • Accept varied solutions/representations

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3. **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

Students:	Teachers:
• Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying and probing questions	• Provide opportunities for students to listen to or read the conclusions and arguments of others • Establish and facilitate a safe environment for discussion • Ask clarifying and probing questions • Avoid giving too much assistance (e.g., providing answers or procedures)

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4. **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

Students:	Teachers:
• Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying questions	• **Allow time for the process to take place (model, make graphs, etc.)** • **Model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written)** • **Make appropriate tools available** • **Create an emotionally safe environment where risk taking is valued** • **Provide meaningful, real world, authentic, performance-based tasks (non traditional work problems)**

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5. **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.

Students:	Teachers:
• Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding	• Make appropriate tools available for learning (calculators, concrete models, digital resources, pencil/paper, compass, protractor, etc.) • Use tools with their instruction

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6. **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

Students:	Teachers:
• Calculate accurately and efficiently • Explain their thinking using mathematics vocabulary • Use appropriate symbols and specify units of measure	• Recognize and model efficient strategies for computation • Use (and challenging students to use) mathematics vocabulary precisely and consistently

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7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.

Students:	Teachers:
• Look for, develop, and generalize relationships and patterns • Apply reasonable thoughts about patterns and properties to new situations	• Provide time for applying and discussing properties • Ask questions about the application of patterns • Highlight different approaches for solving problems

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8. **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Students:	Teachers:
• Look for methods and shortcuts in patterns and repeated calculations • Evaluate the reasonableness of results and solutions	• Provide tasks and problems with patterns • Ask about possible answers before, and reasonableness after computations

4th Grade – CCSS for Mathematics

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4th Grade – CCSS for Mathematics

Grade 4 Overview

Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

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CCSS Where to Focus Grade 4 Mathematics
Not all of the content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have a greater emphasis is not to say that anything in the standards can be safely neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 4

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters □ Supporting Clusters ○ Additional Clusters

4.OA.A	■	Use the four operations with whole numbers to solve problems.
4.OA.B	□	Gain familiarity with factors and multiples.
4.OA.C	○	Generate and analyze patterns.
4.NBT.A	■	Generalize place value understanding for multi-digit whole numbers.
4.NBT.B	■	Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NF.A	■	Extend understanding of fraction equivalence and ordering.
4.NF.B	■	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.C	■	Understand decimal notation for fractions, and compare decimal fractions.
4.MD.A	□	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD.B	□	Represent and interpret data.
4.MD.C	○	Geometric measurement: understand concepts of angle and measure angles.
4.G.A	○	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

REQUIRED FLUENCIES FOR GRADE 4

4.NBT.B.4	Add/subtract within 1,000,000

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

4th Grade – CCSS for Mathematics

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4th Grade – CCSS for Mathematics

4.OA.A Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Essential Skills and Concepts:
- ☐ Add fluently
- ☐ Multiply fluently
- ☐ Convert verbal sentences into mathematical equations

Question Stems and Prompts:
- ✓ Write and draw a model of the equation I am about to read to you.
- ✓ What is another way that we could have written this equation and still have a correct solution?
- ✓ How are _______ alike? Different?

Vocabulary	**Spanish Cognates**
Tier 2	
• comparison	comparación
• represent	representar
• interpret	interpretar
Tier 3	
• multiplicative comparison	comparación multiplicativa
• additive comparison	comparación aditivo
• multiplication equation	ecuación multiplicación
• solution	solución

Standards Connections
4.OA.1 → 4.OA.2

4.OA.1 Examples:

Example: Multiplicative Comparison Problems.
Unknown Product: "Sally is 5 years old. Her mother is 8 times as old as Sally is. How old is Sally's mother?" This problem takes the form $a \times b = ?$, where the factors are known but the product is unknown.
Unknown Factor (Group Size Unknown): "Sally's mother is 40 years old. That is 8 times as old as Sally is, How old is Sally?" This problem takes the form $a \times ? = p$, where the product is known, but the quantity being multiplied to become bigger, is unknown.
Unknown Factor 2 (Number of Groups Unknown): "Sally's mother is 40 years old. Sally is 5 years old. How many times older than Sally is this?" This problem takes the form $? \times b = p$, where the product is known but the multiplicative factor, which does the enlarging in this case, is unknown.

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4th Grade – CCSS for Mathematics

4.OA.A.1

Standard Explanation

In earlier grades students focused on addition and subtraction, and worked with additive comparison problems (e.g., what amount would be added to one quantity in order to result in the other: bigger quantity = smaller quantity + difference), in grade four students compare quantities multiplicatively for the first time

In a multiplicative comparison problem, the underlying structure is that a factor multiplies one quantity to result in the other (e.g., *b* is *n* times as much as *a*, represented by $n \times a = b$). Students interpret a multiplication equation as a comparison and solve word problems involving multiplicative comparison (4.OA.1-2▲) and should be able to identify and verbalize all three quantities involved: which quantity is being multiplied (the smaller quantity), which number tells how many times, and which number is the product (the bigger quantity). Teachers should be aware that students often have difficulty with understanding the order and meaning of numbers in multiplicative comparison problems, and so special attention should be paid to understanding these types of problem situations (MP.1) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Illustrative Tasks:

- Thousands and Millions of Fourth Graders https://www.illustrativemathematics.org/content-standards/4/OA/A/1/tasks/1808

There are almost 40 thousand fourth graders in Mississippi and almost 400 thousand fourth graders in Texas. There are almost 4 million fourth graders in the United States.

We write 4 million as 4,000,000. How many times more fourth graders are there in Texas than in Mississippi? How many times more fourth graders are there in the United States than in Texas? Use the approximate populations listed above to solve.

There are about 4 thousand fourth graders in Washington, D.C. How many times more fourth graders are there in the United States than in Washington, D.C.?

- Threatened and Endangered https://www.illustrativemathematics.org/content-standards/4/OA/A/1/tasks/1809

Maned wolves are a threatened species that live in South America. People estimate that there are about 24,000 of them living in the wild.

The dhole is an endangered species that lives in Asia. People estimate there are ten times as many maned wolves as dholes living in the wild.

About how many dholes are there living in the wild?

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4th Grade – CCSS for Mathematics

4.OA.A Use the four operations with whole numbers to solve problems.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Essential Skills and Concepts:

- ☐ Subtract fluently
- ☐ Add fluently
- ☐ Multiply fluently
- ☐ Divide whole numbers

Question Stems and Prompts:

- ✓ What is the difference between "more than" and "__times".
- ✓ Distinguish multiplicative comparison and additive comparison.
- ✓ Model the multiplicative comparison and the additive comparison.
- ✓ Using tape diagrams model ______ "less than"

Vocabulary	**Spanish Cognates**
Tier 2	
• comparison	comparación
• represent	representar
• model	modelo
• interpret	interpretar
Tier 3	
• multiplicative comparison	comparación multiplicativa
• additive comparison	comparación aditivo
• division comparison	comparación de divisas
• solution	solución

Standards Connections

4.OA.2 → 4.NF.1, 4.NF.4a, 4.MD.1

4.OA.2 Examples:

Example: Multiplicative Comparison Problems 4.OA.2▲

Unknown Product: "Sally is 5 years old. Her mother is 8 times as old as Sally is. How old is Sally's mother?" This problem takes the form $a \times b = ?$, where the factors are known but the product is unknown.

Unknown Factor (Group Size Unknown): "Sally's mother is 40 years old. That is 8 times as old as Sally is. How old is Sally?" This problem takes the form $a \times ? = p$, where the product is known, but the quantity being multiplied is unknown.

Unknown Factor 2 (Number of Groups Unknown): "Sally's mother is 40 years old. Sally is 5 years old. How many times older than Sally is this?" This problem takes the form $? \times b = p$, where the product is known but the multiplicative factor, which does the enlarging in this case, is unknown.

Adapted from Kansas Association of Teachers of Mathematics (KATM) 2012, 4th Grade Flipbook.

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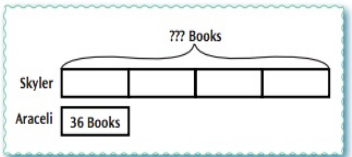
4.OA.A.2

Standard Explanation
This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems.

Examples: Using Tape Diagrams to Represent Multiplication "Compare" Problems 4.OA.2▲

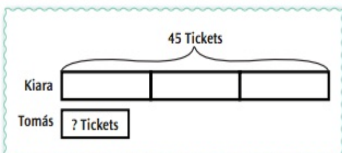
Unknown Product: "Skyler has 4 times as many books as Araceli. If Araceli has 36 books, how many books does Skyler have?"

Solution: If we represent the number of books that Araceli has with a piece of tape, then the number of books Skyler has is represented by 4 pieces of tape of the same size. Students can represent this as 4 × 36 = □.



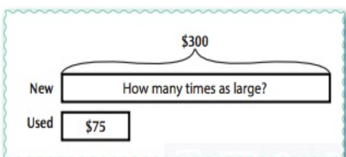
Unknown Factor (Group Size Unknown): "Kiara sold 45 tickets to the school play, which is 3 times as many as the number of tickets sold by Tomás. How many tickets did Tomás sell?"

Solution: The number of tickets Kiara sold (the *product*) is known and is represented by 3 pieces of tape. The number of tickets Tomás sold would be represented by one piece of tape. This representation helps students see that the equations 3 × □ = 45 or 45 ÷ 3 = □ represent the problem.



Unknown Factor (Number of Groups Unknown): "A used bicycle costs $75; a new one costs $300. How many times as much does the new bike cost compared with the used bike?"

Solution: The student represents the cost of the used bike with a piece of tape and decides how many pieces of this tape will make up the cost of the new bike. The representation leads to the equations □ × 75 = 300 and 300 ÷ 75 = □.



Adapted from KATM 2012, 4th Grade Flipbook.

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

Illustrative Task:

- Comparing Money Raised
  https://www.illustrativemathematics.org/content-standards/4/OA/A/2/tasks/263

a. Helen raised $12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year?

b. Sandra raised $15 for the PTA and Nita raised $45. How many times as much money did Nita raise as compared to Sandra?

c. Luis raised $45 for the animal shelter, which was 3 times as much money as Anthony raised. How much money did Anthony raise?

4th Grade – CCSS for Mathematics

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4.OA.A Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Essential Skills and Concepts:
- ☐ Add/Subtract Fluently
- ☐ Multiply/Divide Fluently
- ☐ Understand number order
- ☐ Estimation

Question Stems and Prompts:
- When do we round a number up?
- When do we round a number down?
- Why is rounding helpful?
- ….is closer to…
- …is further from…on a number line

Vocabulary	**Spanish Cognates**
Tier 2	
• multiple	múltiple
• round	redondear
• variable	variable
Tier 3	
• estimate	estimar / estimación
• quotient	cociente
• divisor	divisor

Standards Connections
4.OA.3 → 4.MD.2

4.OA.3 Example:

1.	"There are 146 students going on a field trip. If each bus held 30 students, how many buses are needed?" *Solution:* Since $150 \div 30 = 5$, it seems like there should be around 5 buses. When we try to divide 146 by 30, we get 4 groups with 26 leftover. This means that $146 = 4 \times 30 + 26$. There are 4 filled with 30 students, with a fifth bus holding only 26 students. (In this case, one more than the quotient is the answer.)
2.	"Suppose that 250 pencils were distributed equally among 33 students for a geometry project. What is the largest number of pencils each student can receive?" *Solution:* Since $240 \div 30 = 8$, it seems like each student should receive close to 8 pencils. When we divide 250 by 33, we get 7 with a remainder of 19. This means that $250 = 33 \times 7 + 19$. This tells us that each student can have 7 pencils with 19 leftover for the teacher to hold on to.
3.	"Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each pack. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?" *Solution:* "First, I multiplied 3 packs by 6 bottles per pack which equals 18 bottles. Then I multiplied 6 packs by 6 bottles per pack which is 36 bottles. I know 18 plus 36 is around 50. Since we're trying to get to 300, we'll need about 250 more bottles."

4th Grade – CCSS for Mathematics

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4.OA.3 Example:

1.	"There are 146 students going on a field trip. If each bus held 30 students, how many buses are needed?" *Solution:* Since $150 \div 30 = 5$, it seems like there should be around 5 buses. When we try to divide 146 by 30, we get 4 groups with 26 leftover. This means that $146 = 4 \times 30 + 26$. There are 4 filled with 30 students, with a fifth bus holding only 26 students. (In this case, one more than the quotient is the answer.)
2.	"Suppose that 250 pencils were distributed equally among 33 students for a geometry project. What is the largest number of pencils each student can receive?" *Solution:* Since $240 \div 30 = 8$, it seems like each student should receive close to 8 pencils. When we divide 250 by 33, we get 7 with a remainder of 19. This means that $250 = 33 \times 7 + 19$. This tells us that each student can have 7 pencils with 19 leftover for the teacher to hold on to.
3.	"Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each pack. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?" *Solution:* "First, I multiplied 3 packs by 6 bottles per pack which equals 18 bottles. Then I multiplied 6 packs by 6 bottles per pack which is 36 bottles. I know 18 plus 36 is around 50. Since we're trying to get to 300, we'll need about 250 more bottles."

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4th Grade – CCSS for Mathematics

4.OA.A.3

Standard Explanation
As students compute and interpret multi-step problems with remainders, they also reinforce important mathematical practices as they make sense of the problem and reason about how the context is connected to the four operations (MP.1, MP.2).

Additionally, students solve multi-step word problems using the four operations, including problems in which remainders must be interpreted. (4.OA.3▲). Students use estimation to solve problems. They identify when estimation is appropriate, determine the level of accuracy needed to solve a problem and select the appropriate method of estimation. This gives rounding usefulness, rather than making rounding a separate topic that is covered arbitrarily.

Common Misconceptions

- Teachers may try to help their students by telling them that multiplying two numbers in a multiplicative comparison situation always makes the product *bigger.* While this is true with whole numbers greater than 1, it is *not* true when one of the factors is a fraction smaller than 1 (or when one of the factors is negative), something students will encounter in later grades. Teachers should be careful to emphasize that multiplying by a number *greater than 1* results in a product larger than the original number (**4.OA.1–2▲**).
- Students might be confused by the difference between 6 more than a number (additive) and 6 times a number (multiplicative). For example, using 18 and 6, a question could be "How much more is 18 than 6?" Thinking multiplicatively, the answer is 3; however, thinking additively, the answer is 12 (adapted from KATM 2012, 4th Grade Flipbook).
- It is common practice when dividing numbers to write, for example, $250 \div 33 = 7R19$. Although this notation has been used for quite some time, it obscures the relationship between the numbers in the problem. When students find fractional answers, the correct equation for the present example becomes $250 \div 33 = 7\frac{19}{33}$. It is more accurate to write the answer in words, such as by saying, "When we divide 250 by 33, the quotient is 7 with 19 left over," or to write the equation as $250 = 33 \times 7 + 19$ (see standard **4.NBT.6▲**).

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

Illustrative Tasks:

- Karl's Garden
  https://www.illustrativemathematics.org/content-standards/4/OA/A/3/tasks/876

Karl's rectangular vegetable garden is 20 feet by 45 feet, and Makenna's is 25 feet by 40 feet. Whose garden is larger in area?

- Carnival Tickets
  https://www.illustrativemathematics.org/content-standards/4/OA/A/3/tasks/1289

Every year a carnival comes to Hallie's town. The price of tickets to ride the rides has gone up every year.

Year	Ticket Price
2008	$2.00
2009	$2.50
2010	$3.00
2011	$3.50
2012	$4.00

4.OA.A.3

Standard Explanation
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- Students might be confused by the difference between 6 more than a number (additive) and 6 times a number (multiplicative). For example, using 18 and 6, a question could be "How much more is 18 than 6?" Thinking multiplicatively, the answer is 3; however, thinking additively, the answer is 12 (adapted from KATM 2012, 4th Grade Flipbook).
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4.OA.B Gain familiarity with factors and multiples.

4.OA.4 Gain familiarity with factors and multiples. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Essential Skills and Concepts:

- ☐ Counting by ones
- ☐ Multiples and multiplying
- ☐ Factors of a number between 1-100
- ☐ Prime Numbers and Composite Numbers

Question Stems and Prompts:

- List all the factors of … (e.g. 2x1, 2x2, 2x3, 2x4…)
- What are some things you notice about…
- Why do some numbers come up multiple times and others only once?

Vocabulary	**Spanish Cognates**
Tier 3	
• factor	factor
• multiple	múltiple
• prime	primo
• composite	compuesto
• reverse pairs	pares inversos

Standards Connections

4.OA.4 ← 3.OA.7

4.OA.4 Example:

Common Misconceptions.

- Students may think the number 1 is a prime number or that all prime numbers are odd numbers (counterexample: 2 has only 2 factors—1 and 2).
- When listing multiples of numbers students may not list the number itself. Students should be reminded that the smallest multiple is the number itself.
- Students may think larger numbers have more factors.

Having students share all factor pairs and how they found them will help students avoid some of these misconceptions (Adapted from KATM 4th FlipBook 2012).

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Standard Explanation
At grade four, students find all factor pairs for whole numbers in the range 1–100. Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in grade six.

Students extend the idea of decomposition to multiplication and learn to use the term multiple. Any whole number is a multiple of each of its factors. For example, 21 is a multiple of 3 and a multiple of 7 because 21 = 3× 7. A number can be multiplicatively decomposed into equal groups (e.g., 3 equal groups of 7) and expressed as a product of these two factors (called factor pairs). The only factors for a prime number are 1 and the number itself. A composite number has two or more factor pairs. The number 1 is neither prime nor composite. To find all factor pairs for a given number, students need to search systematically—by checking if 2 is a factor, then 3, then 4, and so on, until they start to see a "reversal" in the pairs. For example, after finding the pair 6 and 9 for 54, students will next find the reverse pair, 9 and 6 (adapted from the University of Arizona [UA] Progressions Documents for the Common Core Math Standards 2011a).

Common Misconceptions

- Students may think the number 1 is a prime number or that all prime numbers are odd numbers. (*Counterexample:* 2 has only two factors—1 and 2—and is therefore prime.)
- When listing multiples of numbers, students may omit the number itself. Students should be reminded that the smallest multiple is the number itself.
- Students may think larger numbers have more factors. (*Counterexample:* 98 has six factors: 1, 2, 7, 14, 49, and 98; 36 has nine factors: 1, 2, 3, 4, 6, 9, 12, 18, and 36.)

Having students share all factor pairs and explain how they found them will help students avoid some of these misconceptions.

Adapted from KATM 2012, 4th Grade Flipbook.

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

4th Grade – CCSS for Mathematics

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4th Grade – CCSS for Mathematics

4.OA.C Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Essential Skills and Concepts:
- ☐ Multiply fluently
- ☐ Count by multiples/factors
- ☐ Generate a pattern

Question Stems and Prompts:
- ✓ What shape should come next?
- ✓ Find the next … shape(s) in the pattern.
- ✓ How can we find the 100th shape in the pattern?
- ✓ What did you observe about the pattern?
- ✓ Can you predict the outcome if….?

Vocabulary	**Spanish Cognates**
Tier 2	
• informally	informalmente
• pattern	patrón
• factors	factores
• multiple	múltiple
Tier 3	
• composite Numbers	números compuestos

Standards Connections
4.OA.5 ← 3.OA.9

4.OA.5 Example:
For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and then reason about how the dots are organized in the design to determine the total number of dots in the 100th design. (MP.2, MP.4, MP.5, MP.7) (Adapted from Progressions K-5 CC and OA 2011).

Growing Patterns available at
http://illuminations.nctm.org/LessonDetail.aspx?ID=U103
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4th Grade – CCSS for Mathematics

4.OA.C.5
Standard Explanation
Understanding patterns is fundamental to algebraic thinking. In grade four students generate and analyze number and shape patterns that follow a given rule.

Students begin by reasoning about patterns, connecting a rule for a given pattern with its sequence of numbers or shapes. A pattern is a sequence that repeats or evolves in a predictable process over and over. A rule dictates what that process will look like. Patterns that consist of repeated sequences of shapes or growing sequences of designs can be appropriate for the grade. For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and then reason about how the dots are organized in the design to determine the total number of dots in the 100th design (MP.2, MP.4, MP.5, MP.7) [adapted from UA Progressions Documents 2011a]. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Operations and Algebraic Thinking Progression:
For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and they could reason about how the dots are organized in the design to determine the total number of dots in the 100th design. In examining numerical sequences, fourth graders can explore rules of repeatedly adding the same whole number or repeatedly multiplying by the same whole number. Properties of repeating patterns of shapes can be explored with division. For example, to determine the 100th shape in a pattern that consists of repetitions of the sequence "square, circle, triangle," the fact that when we divide 100 by 3 the whole number quotient is 33 with remainder 1 tells us that after 33 full repeats, the 99^{th} shape will be a triangle (the last shape in the repeating pattern), so the 100th shape is the first shape in the pattern, which is a square. Notice that the standards do not require students to infer or guess the underlying rule for a pattern, but rather ask them to generate a pattern from a given rule and identify features of the given pattern (K, Counting and Cardinality; K – 5 Operations and Algebraic Thinking, May 29, 2011 http://ime.math.arizona.edu/progressions/).

Illustrative Task:
- Double Plus One
  https://www.illustrativemathematics.org/illustrations/487

a. The table below shows a list of numbers. For every number listed in the table, multiply it by 2 and add 1. Record the result on the right.

number	double the number plus one
0	
1	
2	

4.OA.C.5
Standard Explanation
Understanding patterns is fundamental to algebraic thinking. In grade four students generate and analyze number and shape patterns that follow a given rule.

Students begin by reasoning about patterns, connecting a rule for a given pattern with its sequence of numbers or shapes. A pattern is a sequence that repeats or evolves in a predictable process over and over. A rule dictates what that process will look like. Patterns that consist of repeated sequences of shapes or growing sequences of designs can be appropriate for the grade. For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and then reason about how the dots are organized in the design to determine the total number of dots in the 100th design (MP.2, MP.4, MP.5, MP.7) [adapted from UA Progressions Documents 2011a]. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

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4.NBT.A Generalize place value understanding for multi-digit whole numbers.

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.

Essential Skills and Concepts:
- ☐ Multiplication/Division Fluency
- ☐ Multiply by 10
- ☐ Divide by 10
- ☐ Understand place value

Question Stems and Prompts:
- ✓ What happens if you multiply by 10?
- ✓ What happens if you divide by 10?
- ✓ Compare 1-10-100

Vocabulary	**Spanish Cognates**
Tier 2	
• multiples	múltiples
• more than	más que
• less than	
Tier 3	
• tens	
• hundreds	
• thousands	
• tenths	
• hundredths	
• whole number	
• place value	el valor de posición

Standards Connections
4.NBT.1 → 4.NBT.2, 4.NBT.3, 4.NBT.4, 4.NBT.5, 4.NBT.6

4th Grade – CCSS for Mathematics

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Standards Connections
4.NBT.1 → 4.NBT.2, 4.NBT.3, 4.NBT.4, 4.NBT.5, 4.NBT.6

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4.NBT.A.1

Standard Explanation

In grade four, students extend their work in the base-ten number system and generalize previous place-value understanding to multi-digit whole numbers (less than or equal to 1,000,000).

Students read, write, and compare numbers based on the meaning of the digits in each place (4.NBT.1–2). In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Students can come to see and understand that multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left (adapted from UA Progressions Documents 2012b). Students can develop their understanding of millions by using a place-value chart to understand the pattern of times ten in the base-ten system; for example, 20 hundreds can be bundled into 2 thousands.

Students need multiple opportunities to use real-world contexts to read and write multi-digit whole numbers. As they extend their understanding of numbers to 1,000,000, students reason about the magnitude of digits in a number and analyze the relationships of numbers. They can build larger numbers by using graph paper and labeling examples of each place with digits and words (e.g., 10,000 and ten thousand).

To read and write numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (e.g., thousand, million). Layered place-value cards such as those used in earlier grades can be put on a frame with the base-thousand units labeled below. Then cards that form hundreds, tens, and ones can be placed on each section and the name read off using the card values followed by the word million, then thousand, then the silent ones (MP.2, MP.3, MP.8) (K, Counting and Cardinality; K – 5 Operations and Algebraic Thinking, May 29, 2011 http://ime.math.arizona.edu/progressions/). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Illustrative Tasks:

- Thousands and Millions of Fourth Graders https://www.illustrativemathematics.org/illustrations/1808

There are almost 40 thousand fourth graders in Mississippi and almost 400 thousand fourth graders in Texas. There are almost 4 million fourth graders in the United States.

We write 4 million as 4,000,000. How many times more fourth graders are there in Texas than in Mississippi? How many times more fourth graders are there in the United States than in Texas? Use the approximate populations listed above to solve.

There are about 4 thousand fourth graders in Washington, D.C. How many times more fourth graders are there in the United States than in Washington, D.C.?

4th Grade – CCSS for Mathematics

4.NBT.A.1

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4.NBT.A Generalize place value understanding for multi-digit whole numbers.

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

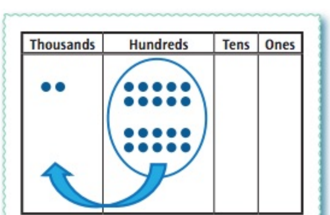
Essential Skills and Concepts:
- ☐ Number order
- ☐ Place value

Question Stems and Prompts:
- ✓ Which number is larger?
- ✓ Which number is smaller?
- ✓ How do you know?
- ✓ Place these numbers on a number line.
- ✓ What is the order?

Vocabulary	**Spanish Cognates**
Tier 2	
• More than	más que
• Less than	
Tier 3	
• Equal	igual
• Expanded Form	forma expandida
• Equal to	igual a

Standards Connections
4.NBT.2 → 4.NBT.3

4.NBT.2 Examples:



2 thousands = 20 hundreds

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones
						4
					4	0
				4	0	0
			4	0	0	0
		4	0	0	0	0
	4	0	0	0	0	0
	4	4	4	4	4	4

"Four hundred forty-four thousand, four hundred forty-four"

4th Grade – CCSS for Mathematics

4.NBT.A Generalize place value understanding for multi-digit whole numbers.

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

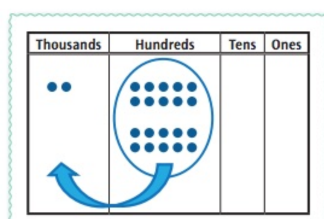
Essential Skills and Concepts:
☐ Number order
☐ Place value

Question Stems and Prompts:
✓ Which number is larger?
✓ Which number is smaller?
✓ How do you know?
✓ Place these numbers on a number line.
✓ What is the order?

Vocabulary / Spanish Cognates
Tier 2
• More than — más que
• Less than
Tier 3
• Equal — igual
• Expanded Form — forma expandida
• Equal to — igual a

Standards Connections
4.NBT.2 → 4.NBT.3

4.NBT.2 Examples:


2 thousands = 20 hundreds

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones
						4
					4	0
				4	0	0
			4	0	0	0
		4	0	0	0	0
	4	0	0	0	0	0
	4	4	4	4	4	4

"Four hundred forty-four thousand, four hundred forty-four"

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4.NBT.A.2

Standard Explanation
In grade four, students extend their work in the base-ten number system and generalize previous place-value understanding to multi-digit whole numbers (less than or equal to 1,000,000).

Students read, write, and compare numbers based on the meaning of the digits in each place (4.NBT.1–2). In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Students can come to see and understand that multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left (adapted from UA Progressions Documents 2012b). Students can develop their understanding of millions by using a place-value chart to understand the pattern of times ten in the base-ten system; for example, 20 hundreds can be bundled into 2 thousands.

Students need multiple opportunities to use real-world contexts to read and write multi-digit whole numbers. As they extend their understanding of numbers to 1,000,000, students reason about the magnitude of digits in a number and analyze the relationships of numbers. They can build larger numbers by using graph paper and labeling examples of each place with digits and words (e.g., 10,000 and ten thousand).

To read and write numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (e.g., thousand, million). Layered place-value cards such as those used in earlier grades can be put on a frame with the base-thousand units labeled below. Then cards that form hundreds, tens, and ones can be placed on each section and the name read off using the card values followed by the word million, then thousand, then the silent ones (MP.2, MP.3, MP.8) (K, Counting and Cardinality; K – 5 Operations and Algebraic Thinking, May 29, 2011 http://ime.math.arizona.edu/progressions/). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Number and Operation Base Ten Progression:
To read numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (thousand, million, billion, trillion, etc.). Thus, 457,000 is read "four hundred fifty seven thousand. The same methods students used for comparing and rounding numbers in previous grades apply to these numbers, because of the uniformity of the base-ten system. **(**K – 5, Number and Operations in Base Ten, April 21, 2012 http://ime.math.arizona.edu/progressions/)

4th Grade – CCSS for Mathematics

4.NBT.A.2

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4.NBT.A Generalize place value understanding for multi-digit whole numbers.

4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

Essential Skills and Concepts:
- ☐ Understand place value
- ☐ Round up
- ☐ Round down
- ☐ Estimation

Question Stems and Prompts:
- ✓ What is the next closest …
- ✓ ___ is closer to___ than___
- ✓ I rounded up/down because…
- ✓ Estimate….

Vocabulary	**Spanish Cognates**
Tier 2	
• round	redondear
• round up	
• round down	

Standards Connections
4.NBT.3 → 4.OA.3

4.NBT.3 Example:

Example: Rounding Numbers in Context 4.NBT.3▲ (MP.4)

The population of the fictional Midtown, USA, was last recorded as 76,398. The city council wants to round the population to the nearest thousand for a business brochure. What number should they round the population to?

Solution: When students represent numbers stacked vertically, they can see the relationships between the numbers more clearly. Students might think: "I know the answer is either 76,000 or 77,000. If I write 76,000 below 76,398 and 77,000 above it, I can see that the midpoint is 76,500, which is *above* 76,398. This tells me they should round the population to 76,000."

77,000
76,398
76,000

Adapted from ADE 2010.

4th Grade – CCSS for Mathematics

4.NBT.A Generalize place value understanding for multi-digit whole numbers.

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Essential Skills and Concepts:
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Question Stems and Prompts:
✓ What is the next closest …
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77,000
76,398
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4.NBT.A.3

Standard Explanation
This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Grade-four students build on the grade-three skill of rounding to the nearest 10 or 100 to round multi-digit numbers and to make reasonable estimates of numerical values. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Progression Information:
Fourth grade students build on the grade three skill of rounding to the nearest 10 or 100 to round multi-digit numbers and to make reasonable estimates of numerical values. **(**K – 5, Number and Operations in Base Ten, April 21, 2012 http://ime.math.arizona.edu/progressions/)

Illustrative Tasks:

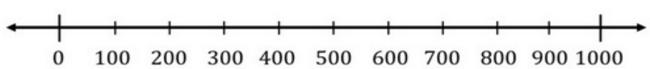
- Rounding to the Nearest 100 and 1000
  https://www.illustrativemathematics.org/content-standards/4/NBT/A/3/tasks/1806

Plot the following numbers on the number line:

80

328

791



a. Round each number to the nearest 100. How can you see this on the number line?

b. Round each number to the nearest 1000. How can you see this on the number line?

- Rounding to the Nearest 1000
  https://www.illustrativemathematics.org/content-standards/4/NBT/A/3/tasks/1807

The tick marks on the number line are evenly spaced. Label them.

Plot the following numbers on the number line:

85

940

2,316

5,090

7,784

Round each number to the nearest 1000. Explain how you can tell which thousand each number will round to by looking at the number line.

4th Grade – CCSS for Mathematics

4.NBT.A.3

Standard Explanation
This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

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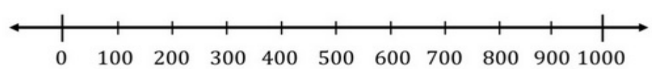

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b. Round each number to the nearest 1000. How can you see this on the number line?

- Rounding to the Nearest 1000
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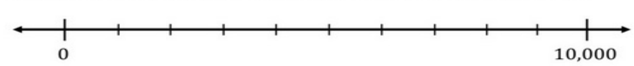

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4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Essential Skills and Concepts:
- ☐ Addition
- ☐ Subtraction
- ☐ Place Value
- ☐ Expanded form

Question Stems and Prompts:
- ✓ Why is it important that we line up our number according to place value?
- ✓ When we have more than 9 units we have to convert.
- ✓ What place value is _______ in?
- ✓ _______ represents ________ place value
- ✓ Write _________ in the expanded form.

Vocabulary	**Spanish Cognates**
Tier 2	
• convert	convertir
Tier 3	
• ones	unos/unidad
• tens	
• hundreds	
• expanded form	forma expandida
• place value	el valor de posición
• equal to	igual

Standards Connections
4.NBT.4 ← 4.NBT.1

4.NBT.4 Example:

FLUENCY

California's Common Core State Standards for Mathematics (K–6) set expectations for fluency in computations using the standard algorithm (e.g., "*Fluently* add and subtract multi-digit whole numbers using the standard algorithm" [**4.NBT.4▲**]). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. The word *fluent* is used in the standards to mean "reasonably fast and accurate" and possessing the ability to use certain facts and procedures with enough facility that using such knowledge does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of knowing some answers, knowing some answers from patterns, and knowing some answers through the use of strategies.

Adapted from UA Progressions Documents 2011a.

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

4th Grade – CCSS for Mathematics

4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Essential Skills and Concepts:
- ☐ Addition
- ☐ Subtraction
- ☐ Place Value
- ☐ Expanded form

Question Stems and Prompts:
- ✓ Why is it important that we line up our number according to place value?
- ✓ When we have more than 9 units we have to convert.
- ✓ What place value is _______ in?
- ✓ _______ represents ________ place value
- ✓ Write _________ in the expanded form.

Vocabulary	**Spanish Cognates**
Tier 2	
• convert	convertir
Tier 3	
• ones	unos/unidad
• tens	
• hundreds	
• expanded form	forma expandida
• place value	el valor de posición
• equal to	igual

Standards Connections
4.NBT.4 ← 4.NBT.1

4.NBT.4 Example:

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4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Essential Skills and Concepts:
- ☐ Multiply fluently
- ☐ Create groups
- ☐ Place value

Question Stems and Prompts:
- ✓ Model the equation using groups.
- ✓ Create a rectangular array/area model
- ✓ Summarize what you did.
- ✓ How could you organize your array?
- ✓ What was the total?

Vocabulary	**Spanish Cognates**
Tier 2	
• product	producto
Tier 3	
• multiply	multiplicar
• expanded form	forma expandida
• rectangular array	matriz rectangular
• area model	modelo de área

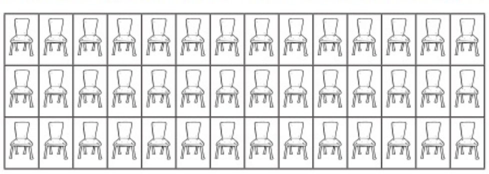
Standards Connections
4.NBT.5 → 4.NBT.6

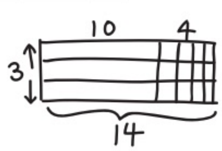
4.NBT.5 Example:

Example: Area Models and Strategies for Multi-Digit Multiplication with a Single-Digit Multiplier 4.NBT.5▲

Chairs are being set up for a small play. There should be 3 rows of chairs and 14 chairs in each row. How many chairs will be needed?

Solution: As in grade three, when students first made the connection between array models and the area model, students might start by drawing a sketch of the situation. They can then be reminded to see the chairs as if surrounded by unit squares and hence a model of a rectangular region. With base-ten blocks or math drawings (**MP.2, MP.5**), students represent the problem and see it broken down into $3\times(10+4)$.





Making a sketch like the one above becomes cumbersome, so students move toward representing such drawings more abstractly, with rectangles, as shown to the right. This builds on the work begun in grade three. Such diagrams help children see the distributive property: "3×14 can be written as $3\times(10+4)$, and I can do the multiplications separately and add the results: $3\times(10+4)=3\times 10+3\times 4$. The answer is $30+12=42$, or 42 chairs."

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

4th Grade – CCSS for Mathematics

4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Essential Skills and Concepts:
☐ Multiply fluently
☐ Create groups
☐ Place value

Question Stems and Prompts:
✓ Model the equation using groups.
✓ Create a rectangular array/area model
✓ Summarize what you did.
✓ How could you organize your array?
✓ What was the total?

Vocabulary Spanish Cognates
Tier 2
• product producto
Tier 3
• multiply multiplicar
• expanded form forma expandida
• rectangular array matriz rectangular
• area model modelo de área

Standards Connections
4.NBT.5 → 4.NBT.6

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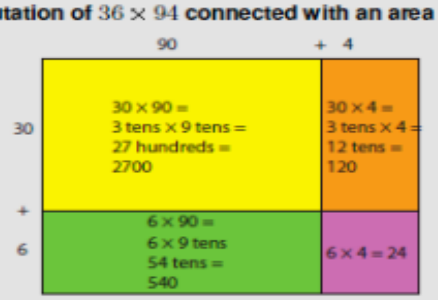
4th Grade – CCSS for Mathematics

4.NBT.B.5

Standard Explanation
At grade four, students become fluent with addition and subtraction with multi-digit whole numbers to 1,000,000 using standard algorithms (4.NBT.4▲). A central theme in multi-digit arithmetic is to encourage students to develop methods they understand and can explain rather than merely following a sequence of directions, rules, or procedures they do not understand. In previous grades, students built a conceptual understanding of addition and subtraction with whole numbers as they applied multiple methods to compute and solve problems. The emphasis in grade four is on the power of the regular one-for-ten trades between adjacent places that let students extend a method they already know to many places. Because students in grades two and three have been using at least one method that will generalize to 1,000,000, this extension in grade four should not take a long time. Thus, students will also have sufficient time for the major new topics of multiplication and division (4.NBT.5–6▲) (CA Mathematics Framework, adopted Nov. 6, 2013).

Number and Operations Base Ten Progression:
In fourth grade, students compute products of one-digit numbers and multi-digit numbers (up to four digits) and products of two digit numbers.4.NBT.5 They divide multi-digit numbers (up to four digits) by one-digit numbers. As with addition and subtraction, students should use methods they understand and can explain. Visual representations such as area and array diagrams that students draw and connect to equations and other written numerical work are useful for this purpose. By reasoning repeatedly about the connection between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities (K – 5, Number and Operations in Base Ten, April 21, 2012 http://ime.math.arizona.edu/progressions/).

Computation of 36 × 94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

4.NBT.B.5

Standard Explanation
At grade four, students become fluent with addition and subtraction with multi-digit whole numbers to 1,000,000 using standard algorithms (4.NBT.4▲). A central theme in multi-digit arithmetic is to encourage students to develop methods they understand and can explain rather than merely following a sequence of directions, rules, or procedures they do not understand. In previous grades, students built a conceptual understanding of addition and subtraction with whole numbers as they applied multiple methods to compute and solve problems. The emphasis in grade four is on the power of the regular one-for-ten trades between adjacent places that let students extend a method they already know to many places. Because students in grades two and three have been using at least one method that will generalize to 1,000,000, this extension in grade four should not take a long time. Thus, students will also have sufficient time for the major new topics of multiplication and division (4.NBT.5–6▲) (CA Mathematics Framework, adopted Nov. 6, 2013).

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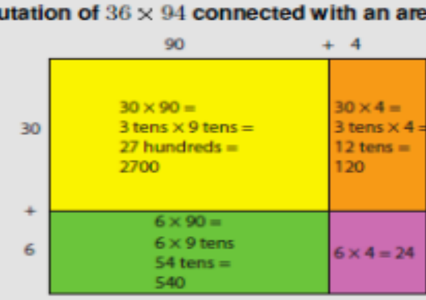

The products of like base-ten units are shown as parts of a rectangular region.

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4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

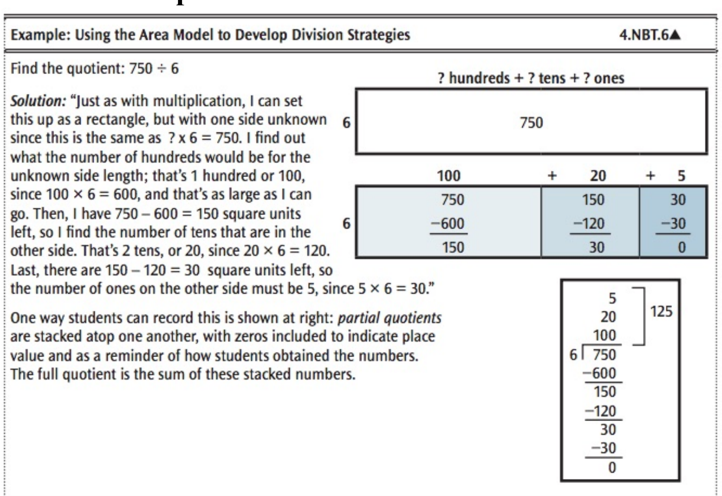
Essential Skills and Concepts:
- ☐ Multiply/Divide Fluently
- ☐ Sketch rectangular arrays
- ☐ Place Value

Question Stems and Prompts:
- ✓ Which number is a dividend?
- ✓ Which number is the divisor?
- ✓ Explain how the dividend and the divisor are different.
- ✓ Sketch a rectangular array to find how many groups of ____ are going to be needed.

Vocabulary	**Spanish Cognates**
Tier 3	
• quotient	cociente
• dividend	divedendo
• divisor	divisor
• rectangular array	matriz rectangular
• equation	ecuación

Standards Connections
4.NBT.6 → 4.OA.3

4.NBT.6 Example:


Example: Using the Area Model to Develop Division Strategies 4.NBT.6▲

Find the quotient: 750 ÷ 6

Solution: "Just as with multiplication, I can set this up as a rectangle, but with one side unknown since this is the same as ? x 6 = 750. I find out what the number of hundreds would be for the unknown side length; that's 1 hundred or 100, since 100 × 6 = 600, and that's as large as I can go. Then, I have 750 – 600 = 150 square units left, so I find the number of tens that are in the other side. That's 2 tens, or 20, since 20 × 6 = 120. Last, there are 150 – 120 = 30 square units left, so the number of ones on the other side must be 5, since 5 × 6 = 30."

One way students can record this is shown at right: *partial quotients* are stacked atop one another, with zeros included to indicate place value and as a reminder of how students obtained the numbers. The full quotient is the sum of these stacked numbers.

(CA Mathematics Framework, adopted Nov. 6, 2013).

4th Grade – CCSS for Mathematics

4.NBT.B Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

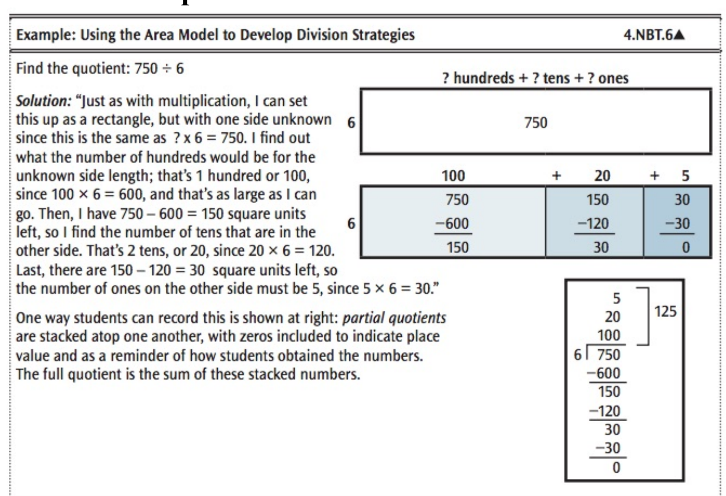
Essential Skills and Concepts:
☐ Multiply/Divide Fluently
☐ Sketch rectangular arrays
☐ Place Value

Question Stems and Prompts:
✓ Which number is a dividend?
✓ Which number is the divisor?
✓ Explain how the dividend and the divisor are different.
✓ Sketch a rectangular array to find how many groups of ____ are going to be needed.

Vocabulary Spanish Cognates
Tier 3
• quotient cociente
• dividend divedendo
• divisor divisor
• rectangular array matriz rectangular
• equation ecuación

Standards Connections
4.NBT.6 → 4.OA.3

4.NBT.6 Example:



(CA Mathematics Framework, adopted Nov. 6, 2013).

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4th Grade – CCSS for Mathematics

4.NBT.B.6

Standard Explanation

In grade four, students extend multiplication and division to include whole numbers greater than 100. Students should use methods they understand and can explain to multiply and divide. The standards call for students to use visual representations such as area and array models that students draw and connect to equations, as well as written numerical work, to support student reasoning and explanation of methods. By reasoning repeatedly about the connections between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities. Students can use area models to represent various multiplication situations. The rows can represent the equal groups of objects in the situation, and students then imagine that the objects lie in the squares forming an array. With larger numbers, such array models become too difficult to draw, so students can make sketches of rectangles and then label the resulting product as the number of things or square units. When area models are used to represent an actual area situation, the two factors are expressed in length units (e.g., *cm*) while the product is in square units (e.g., cm^2).

General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication, but these are cast in terms of division. For example, students may see division problems as knowing the area of a rectangle but not one side length (the quotient), or as finding the size of a group when the number of groups is known (measurement division).

General methods for multi-digit division computation include decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this method relies on the distributive property. This work continues in grade five and culminates in fluency with the standard algorithm in grade six (adapted from PARCC 2012). In grade four, students also find whole-number quotients with remainders (4.NBT.6) and learn the appropriate way to write the result. As students decompose numbers to solve division problems, they also reinforce important mathematical practices such as seeing and making use of structure (MP.7). As they illustrate and explain calculations, they model (MP.4), strategically use appropriate drawings as tools (MP.5), and attend to precision (MP.6) using base-ten units (CA Mathematics Framework, adopted Nov. 6, 2013).

4.NBT.B.6

Standard Explanation

In grade four, students extend multiplication and division to include whole numbers greater than 100. Students should use methods they understand and can explain to multiply and divide. The standards call for students to use visual representations such as area and array models that students draw and connect to equations, as well as written numerical work, to support student reasoning and explanation of methods. By reasoning repeatedly about the connections between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities. Students can use area models to represent various multiplication situations. The rows can represent the equal groups of objects in the situation, and students then imagine that the objects lie in the squares forming an array. With larger numbers, such array models become too difficult to draw, so students can make sketches of rectangles and then label the resulting product as the number of things or square units. When area models are used to represent an actual area situation, the two factors are expressed in length units (e.g., *cm*) while the product is in square units (e.g., cm^2).

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4.NF.A Extend understanding of fractions equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

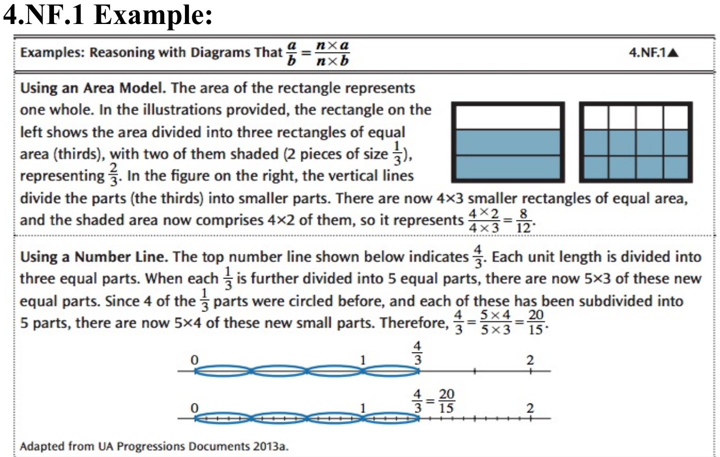
Essential Skills and Concepts:
- ☐ Multiply a number times 1 or n/n.
- ☐ Understand parts/vocabulary of fractions
- ☐ Know that n/n is equal to 1 whole.

Question Stems and Prompts:
- ✓ Label all the parts of a fraction.
- ✓ What does the top number represent?
- ✓ What does the bottom number represent?
- ✓ How are the numerator and denominator similar?
- ✓ How are they different?
- ✓ What do you think it means when the numerator is larger than the denominator?

Vocabulary	**Spanish Cognates**
Tier 2	
• equivalent	equivalente
• simplify	simplificar
Tier 3	
• whole number	
• expanded form	
• numerator	numerador
• denominator	denominador
• equivalent fractions	fracciones equivalentes

Standards Connections
4.NF.1 → 4.NF.2, 4.NF.3a-c

4.NF.1 Example:


Examples: Reasoning with Diagrams That $\frac{a}{b} = \frac{n \times a}{n \times b}$ 4.NF.1▲

Using an Area Model. The area of the rectangle represents one whole. In the illustrations provided, the rectangle on the left shows the area divided into three rectangles of equal area (thirds), with two of them shaded (2 pieces of size $\frac{1}{3}$), representing $\frac{2}{3}$. In the figure on the right, the vertical lines divide the parts (the thirds) into smaller parts. There are now 4×3 smaller rectangles of equal area, and the shaded area now comprises 4×2 of them, so it represents $\frac{4\times2}{4\times3} = \frac{8}{12}$.

Using a Number Line. The top number line shown below indicates $\frac{4}{3}$. Each unit length is divided into three equal parts. When each $\frac{1}{3}$ is further divided into 5 equal parts, there are now 5×3 of these new equal parts. Since 4 of the $\frac{1}{3}$ parts were circled before, and each of these has been subdivided into 5 parts, there are now 5×4 of these new small parts. Therefore, $\frac{4}{3} = \frac{5\times4}{5\times3} = \frac{20}{15}$.

Adapted from UA Progressions Documents 2013a.

4th Grade – CCSS for Mathematics

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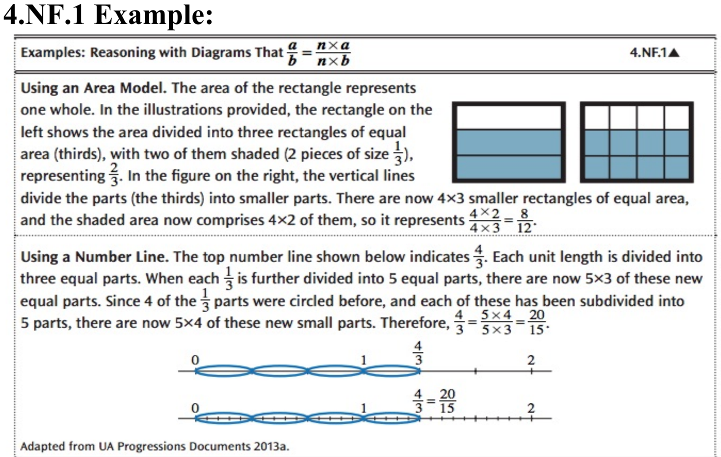
- ☐ Multiply a number times 1 or n/n.
- ☐ Understand parts/vocabulary of fractions
- ☐ Know that n/n is equal to 1 whole.

Question Stems and Prompts:
- ✓ Label all the parts of a fraction.
- ✓ What does the top number represent?
- ✓ What does the bottom number represent?
- ✓ How are the numerator and denominator similar?
- ✓ How are they different?
- ✓ What do you think it means when the numerator is larger than the denominator?

Vocabulary / **Spanish Cognates**
Tier 2: equivalent – equivalente; simplify – simplificar
Tier 3: whole number; expanded form; numerator – numerador; denominator – denominador; equivalent fractions – fracciones equivalentes

Standards Connections
4.NF.1 → 4.NF.2, 4.NF.3a-c

4.NF.1 Example: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Adapted from UA Progressions Documents 2013a.

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4th Grade – CCSS for Mathematics

4.NF.A.1

Standard Explanation

Grade-four students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction (e.g., a b = n a n b, for n ≠ 0). Students use visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size (4.NF.1▲). This property forms the basis for much of the work with fractions in grade four, including comparing, adding, and subtracting fractions and the introduction of finite decimals.

Students use visual models to reason about and explain why fractions are equivalent. Students notice connections between the models and the fractions represented by the models in the way both the parts and wholes are counted, and they begin to generate a rule for writing equivalent fractions. Students also emphasize the inversely related changes: the number of unit fractions becomes larger, but the size of the unit fraction becomes smaller.

Students should have repeated opportunities to use math drawings such as these (and the ones that follow in this chapter) to understand the general method for finding equivalent fractions students may also come to see that the rule works both ways.

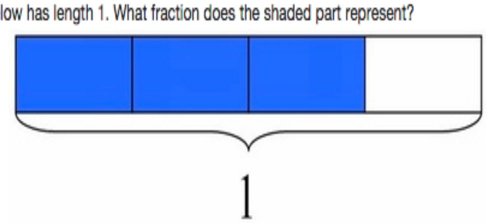
Teachers must be careful to avoid overemphasizing this "simplifying" of fractions, as there is no mathematical reason for doing so—although, depending on the problem context, one form (*renamed* or not *renamed*) may be more desirable than another. In particular, teachers should avoid using the term *reducing* fractions for this process, as the value of the fraction itself is *not* being reduced. A more neutral term, such as renaming (which hints at these fractions being different names for the same amount) allows teachers to refer to this strategy with less potential for student misunderstanding (CA Mathematics Framework, adopted Nov. 6, 2013).

Illustrative Task:

- Explaining Fraction Equivalence with Pictures

http://www.illustrativemathematics.org/illustrations/743

a. The rectangle below has length 1. What fraction does the shaded part represent?



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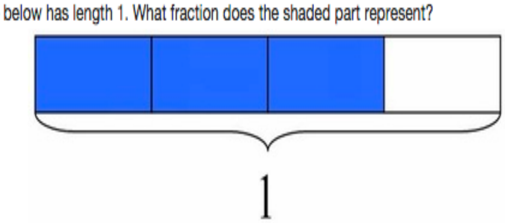

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4.NF.A Extend understanding of fractions equivalence and ordering.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Essential Skills and Concepts:
- ☐ Number order
- ☐ Numerator and denominator
- ☐ Converting fractions

Question Stems and Prompts:
- ✓ Place ____ fraction on a number line
- ✓ Which fraction is larger?
- ✓ Which fraction is smaller?
- ✓ Are the fractions equivalent?
- ✓ Model/tell me an example to prove this.

Vocabulary	**Spanish Cognates**
Tier 2	
• more than	más que
• less than	
• equal	igual
• compare	comparar
• convert	convertir
Tier 3	
• equivalent fractions	fracciones equivalentes
• equal to	igual a

Standards Connections
4.NF.2 → 4.NF.7

Illustrative Task:
- Listing Fractions in Increasing Size
  https://www.illustrativemathematics.org/content-standards/4/NF/A/2/tasks/811

Order the following fractions from smallest to largest:

$$\frac{3}{8}, \quad \frac{1}{3}, \quad \frac{5}{9}, \quad \frac{2}{5}$$

Explain your reasoning.

4^{th} Grade – CCSS for Mathematics

4.NF.A Extend understanding of fractions equivalence and ordering.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

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- ☐ Number order
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Tier 2	
• more than	más que
• less than	
• equal	igual
• compare	comparar
• convert	convertir
Tier 3	
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• equal to	igual a

Standards Connections
4.NF.2 → 4.NF.7

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4.NF.A Extend understanding of fractions equivalence and ordering.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Standard Explanation
Students apply their new understanding of equivalent fractions to compare two fractions with different numerators and different denominators (4.NF.2 ▲). They compare fractions by using benchmark fractions and finding common denominators or common numerators. Students explain their reasoning and record their results using the symbols >, =, and < (CA Mathematics Framework, adopted Nov. 6, 2013).

Examples: Comparing Fractions	4.NF.2▲
1. Students might compare fractions to benchmark fractions—for example, comparing to $\frac{1}{2}$ when comparing $\frac{3}{8}$ and $\frac{2}{3}$. Students see that $\frac{3}{8} < \frac{4}{8} = \frac{1}{2}$, and that since $\frac{2}{3} = \frac{4}{6}$ and $\frac{4}{6} > \frac{3}{6} = \frac{1}{2}$, it must be true that $\frac{3}{8} < \frac{2}{3}$.	
2. Students compare $\frac{5}{8}$ and $\frac{7}{12}$ by writing them with a common denominator. They find that $\frac{5}{8} = \frac{5 \times 12}{8 \times 12} = \frac{60}{96}$ and $\frac{7}{12} = \frac{7 \times 8}{12 \times 8} = \frac{56}{96}$ and reason therefore that $\frac{5}{8} > \frac{7}{12}$. Notice that students do not need to find the **smallest** common denominator for two fractions; any common denominator will work.	
3. Students can also find a common numerator to compare $\frac{5}{8}$ and $\frac{7}{12}$. They find that $\frac{5}{8} = \frac{5 \times 7}{8 \times 7} = \frac{35}{56}$ and $\frac{7}{12} = \frac{7 \times 5}{12 \times 5} = \frac{35}{60}$. Then they reason that, since parts of size $\frac{1}{56}$ are larger than parts of size $\frac{1}{60}$ when the whole is the same, $\frac{5}{8} > \frac{7}{12}$.	
Adapted from ADE 2010.	

Illustrative Task:
- Using Benchmark Fractions to Compare
  http://www.illustrativemathematics.org/illustrations/812

Melissa gives her classmates the following explanation for why $\frac{1}{5} < \frac{2}{7}$:

I can compare both $\frac{1}{5}$ and $\frac{2}{7}$ to $\frac{1}{4}$.

Since $\frac{1}{5}$ and $\frac{1}{4}$ are unit fractions and fifths are smaller than fourths, I know that $\frac{1}{5} < \frac{1}{4}$.

I also know that $\frac{1}{4}$ is the same as $\frac{2}{8}$, so $\frac{2}{7}$ is bigger than $\frac{1}{4}$.

Therefore $\frac{1}{5} < \frac{2}{7}$.

4th Grade – CCSS for Mathematics

4.NF.A Extend understanding of fractions equivalence and ordering.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Standard Explanation
Students apply their new understanding of equivalent fractions to compare two fractions with different numerators and different denominators (4.NF.2 ▲). They compare fractions by using benchmark fractions and finding common denominators or common numerators. Students explain their reasoning and record their results using the symbols >, =, and < (CA Mathematics Framework, adopted Nov. 6, 2013).

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1. Students might compare fractions to benchmark fractions—for example, comparing to $\frac{1}{2}$ when comparing $\frac{3}{8}$ and $\frac{2}{3}$. Students see that $\frac{3}{8} < \frac{4}{8} = \frac{1}{2}$, and that since $\frac{2}{3} = \frac{4}{6}$ and $\frac{4}{6} > \frac{3}{6} = \frac{1}{2}$, it must be true that $\frac{3}{8} < \frac{2}{3}$.
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Therefore $\frac{1}{5} < \frac{2}{7}$.

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4.NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Essential Skills and Concepts:
- ☐ Converting fractions into common denominators
- ☐ Multiplying fractions
- ☐ Multiplying by a whole number as a fraction (n/n)
- ☐ Addition/Subtraction

Question Stems and Prompts:
- ✓ What fractions make up 3/8? (1/8 + 1/8 +1/8)
- ✓ How many different ways can we decompose 3/8?
- ✓ How many ways can you decompose 2 $^{1}/_{8}$?
- ✓ What does the 2 represent? What would that look like as a fraction?

Vocabulary	**Spanish Cognates**
Tier 2	
• simplify	simplificar
• decompose	descomponer
Tier 3	
• numerator	numerador
• denominator	denominador
• equivalent fractions	fracciones equivalentes
• mixed numbers	numero mezclado

Standards Connections
4.NF.3a-c → 4.NF.3d, 4.NF.5, 4.MD.2
4.NF.3d → 4.MD.2, 4.MD.4

4^{th} *Grade – CCSS for Mathematics*

4.NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
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Essential Skills and Concepts:
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- ☐ Multiplying by a whole number as a fraction (n/n)
- ☐ Addition/Subtraction

Question Stems and Prompts:
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- ✓ How many different ways can we decompose 3/8?
- ✓ How many ways can you decompose 2 $^{1}/_{8}$?
- ✓ What does the 2 represent? What would that look like as a fraction?

Vocabulary	**Spanish Cognates**
Tier 2	
• simplify	simplificar
• decompose	descomponer
Tier 3	
• numerator	numerador
• denominator	denominador
• equivalent fractions	fracciones equivalentes
• mixed numbers	numero mezclado

Standards Connections
4.NF.3a-c → 4.NF.3d, 4.NF.5, 4.MD.2
4.NF.3d → 4.MD.2, 4.MD.4

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4th Grade – CCSS for Mathematics

4.NF.B.3

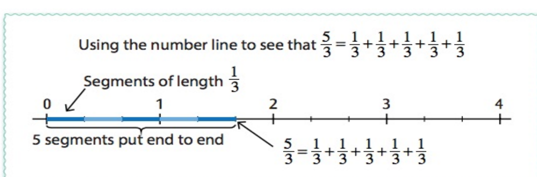
Standard Explanation
In grade four, students extend previous understanding of addition and subtraction of whole numbers to add and subtract fractions with like denominators (4.NF.3▲).

Students begin by understanding a fraction $\frac{a}{b}$ as a sum of the unit fractions $\frac{1}{b}$. In grade three, students a learned that the fraction b represents parts when a whole is broken into *b* equal parts (i.e., parts of 1 size b.) However, in grade four, students connect this understanding of a fraction with the operation of addition; for instance, they see now that if a whole is broken into 4 equal parts and 5 of them are taken, then this is represented by both $\frac{5}{4}$ and the expression $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ (4.NF.3b▲). They experience composing fractions from and decomposing fractions into sums of unit fractions and non-unit fractions in this general way—for example, by seeing $\frac{5}{4}$ also as:

$\boxed{\frac{1}{4}+\frac{1}{4}+\frac{3}{4}}$ or $\boxed{\frac{2}{4}+\frac{3}{4}}$ or $\boxed{\frac{1}{4}+\frac{3}{4}+\frac{1}{4}}$

Students write and use unit fractions while working with standard 4.NF.3b ▲, which supports their conceptual understanding of adding fractions and solving problems (4.NF.3a▲, 4.NF.3d▲). Students write and use unit fractions while decomposing fractions in several ways (4.NF.3b▲). This work helps students understand addition and subtraction of fractions (4.NF.3a▲) and how to solve word problems involving fractions with the same denominator (4.NF.3d ▲). Writing and using unit fractions also helps students avoid the common misconception of adding two fractions by adding their numerators and denominators—for example, erroneously writing. In general, the meaning of addition is the same for both fractions and whole numbers. Students understand addition as "putting together" like units, and they visualize how fractions are built from unit fractions and that a fraction is a sum of unit fractions.

Students may use visual models to support this understanding—for example, showing that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ by using a number line model (MP.1, MP.2, MP.4, MP.6, MP.7) (CA Mathematics Framework, adopted Nov. 6, 2013).



Source: UA Progressions Documents 2013a.

4.NF.B.3

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Students begin by understanding a fraction $\frac{a}{b}$ as a sum of the unit fractions $\frac{1}{b}$. In grade three, students a learned that the fraction b represents parts when a whole is broken into *b* equal parts (i.e., parts of 1 size b.) However, in grade four, students connect this understanding of a fraction with the operation of addition; for instance, they see now that if a whole is broken into 4 equal parts and 5 of them are taken, then this is represented by both $\frac{5}{4}$ and the expression $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ (4.NF.3b▲). They experience composing fractions from and decomposing fractions into sums of unit fractions and non-unit fractions in this general way—for example, by seeing $\frac{5}{4}$ also as:

$\boxed{\frac{1}{4}+\frac{1}{4}+\frac{3}{4}}$ or $\boxed{\frac{2}{4}+\frac{3}{4}}$ or $\boxed{\frac{1}{4}+\frac{3}{4}+\frac{1}{4}}$

Students write and use unit fractions while working with standard 4.NF.3b ▲, which supports their conceptual understanding of adding fractions and solving problems (4.NF.3a▲, 4.NF.3d▲). Students write and use unit fractions while decomposing fractions in several ways (4.NF.3b▲). This work helps students understand addition and subtraction of fractions (4.NF.3a▲) and how to solve word problems involving fractions with the same denominator (4.NF.3d ▲). Writing and using unit fractions also helps students avoid the common misconception of adding two fractions by adding their numerators and denominators—for example, erroneously writing. In general, the meaning of addition is the same for both fractions and whole numbers. Students understand addition as "putting together" like units, and they visualize how fractions are built from unit fractions and that a fraction is a sum of unit fractions.

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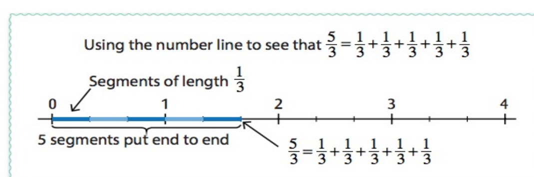

Source: UA Progressions Documents 2013a.

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4th Grade – CCSS for Mathematics

4.NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Essential Skills and Concepts:
- ☐ Multiplication of fractions
- ☐ Decompose fractions

Question Stems and Prompts:
- ✓ How many different ways can we write 5/4? (1/4+1/4+1/4+1/4+1/4 or 1/4 x 5)

Vocabulary	**Spanish Cognates**
Tier 2	
• more	más
• less	
• equal	igual
• more than	más que
• less than	menos que
Tier 3	
• expanded form	forma expandida
• equal to	igual a

Standards Connections
4.NF.4a → 4.NF.4b, 4.NF.4c
4.NF.4b → 4.NF.4c
4.NF.4c → 4.MD.2

4.NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
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Essential Skills and Concepts:
- ☐ Multiplication of fractions
- ☐ Decompose fractions

Question Stems and Prompts:
- ✓ How many different ways can we write 5/4? (1/4+1/4+1/4+1/4+1/4 or 1/4 x 5)

Vocabulary	**Spanish Cognates**
Tier 2	
• more	más
• less	
• equal	igual
• more than	más que
• less than	menos que
Tier 3	
• expanded form	forma expandida
• equal to	igual a

Standards Connections
4.NF.4a → 4.NF.4b, 4.NF.4c
4.NF.4b → 4.NF.4c
4.NF.4c → 4.MD.2

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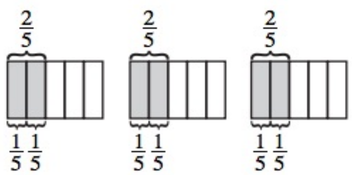
4.NF.B.4

Standard Explanation

In grade three, students learned that 3× 7 can be represented as the total number of objects in 3 groups of 7 objects and that they could solve this by adding 7 + 7 + 7. Fourth-grade students apply this concept to fractions, understanding a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ (4.NF.4a ▲). This understanding is connected with standard 4.NF.3, and students make the shift to see $\frac{5}{3}$ as $5 \times \frac{1}{3}$.

Students then extend this understanding to make meaning of the product of a whole number and a fraction (4.NF.4b ▲)—for example, seeing in the following ways:



$$\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{3\times 2}{5}=\frac{6}{5}$$

Source: UA Progressions Documents 2013a.

Students are also presented with opportunities to work with word problems involving multiplication of a fraction by a whole number to relate situations, models, and corresponding equations (4.NF.4c ▲) (CA Mathematics Framework, adopted Nov. 6, 2013).

Illustrative Task:

- Sugar in six cans of soda
  http://www.illustrativemathematics.org/illustrations/857

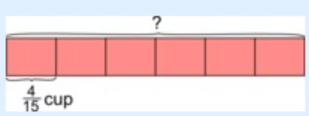
For a certain brand of orange soda, each can contains $\frac{4}{15}$ cup of sugar.

a. How many cups of sugar are there in six cans of this orange soda?

b. Draw a picture representing the answer to (a).

Solution b:

Solution: Using a bar diagram



1 can: $\frac{4}{15}$ cup

6 cans: $6 \times \frac{4}{15} = \frac{6x4}{15} = \frac{24}{15} = 1\frac{9}{15} = 1\frac{3}{5}$ cups

There would be $1\frac{3}{5}$ cups of sugar in 6 cans of soda.

4th Grade – CCSS for Mathematics

4.NF.B.4

Standard Explanation

In grade three, students learned that 3× 7 can be represented as the total number of objects in 3 groups of 7 objects and that they could solve this by adding 7 + 7 + 7. Fourth-grade students apply this concept to fractions, understanding a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ (4.NF.4a ▲). This understanding is connected with standard 4.NF.3, and students make the shift to see $\frac{5}{3}$ as $5 \times \frac{1}{3}$.

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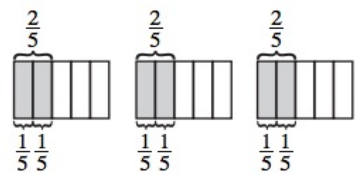

$$\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{3\times 2}{5}=\frac{6}{5}$$

Source: UA Progressions Documents 2013a.

Students are also presented with opportunities to work with word problems involving multiplication of a fraction by a whole number to relate situations, models, and corresponding equations (4.NF.4c ▲) (CA Mathematics Framework, adopted Nov. 6, 2013).

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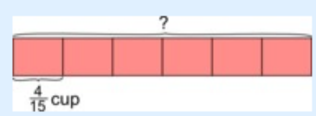

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There would be $1\frac{3}{5}$ cups of sugar in 6 cans of soda.

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4.NF.C Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.4 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

Essential Skills and Concepts:

- ☐ Convert fractions
- ☐ Multiply fractions
- ☐ Create equivalent fractions
- ☐ Add equivalent fractions

Question Stems and Prompts:

✓

Vocabulary	**Spanish Cognates**
Tier 2	
• more	más
• less	
• number	numero
• more than	más que
• less than	menos que
• count	contar / cuenta
• convert	convertir
Tier 3	
• decimal fraction	fracción decimal
• expanded form	forma expandida
• equal to	igual a
• equivalent fraction	fracción equivalente
• tenths	
• hundredths	

Standards Connections

4.NF.5 → 4.NF.6, 4.MD.2

4th Grade – CCSS for Mathematics

4.NF.C Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.4 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

Essential Skills and Concepts:

- ☐ Convert fractions
- ☐ Multiply fractions
- ☐ Create equivalent fractions
- ☐ Add equivalent fractions

Question Stems and Prompts:

✓

Vocabulary	**Spanish Cognates**
Tier 2	
• more	más
• less	
• number	numero
• more than	más que
• less than	menos que
• count	contar / cuenta
• convert	convertir
Tier 3	
• decimal fraction	fracción decimal
• expanded form	forma expandida
• equal to	igual a
• equivalent fraction	fracción equivalente
• tenths	
• hundredths	

Standards Connections

4.NF.5 → 4.NF.6, 4.MD.2

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4.NF.C.5

Standard Explanation
Fourth-grade students develop an understanding of decimal notation for fractions and compare decimal fractions (fractions with a denominator of 10 or 100). This work lays the foundation for performing operations with decimal numbers in grade five. Students learn to add decimal fractions by converting them to fractions with the same denominator (4.NF.5 ▲). For example, students express $\frac{3}{10}$ as $\frac{30}{100}$ before they add $\frac{30}{100} + \frac{4}{100} = \frac{34}{100}$. Students can use graph paper, base-ten blocks, and other place-value models to explore the relationship between fractions with denominators of 10 and 100 (adapted from UA Progressions Documents 2013a) (CA Mathematics Framework, adopted Nov. 6, 2013).

NF Progression Information:
Students in Grade 4 work with fractions having denominators 10 and 100. Because it involves partitioning into 10 equal parts and treating the parts as numbers called one tenth and one hundredth, work with these fractions can be used as preparation to extend the base-ten system to non-whole numbers (Number and Operations – Fractions, 3 – 5, September 19, 2013 http://ime.math.arizona.edu/progressions/).

Illustrative Task:
- Expanded Fractions and Decimals
  http://www.illustrativemathematics.org/illustrations/145

Fraction	Expanded Fraction Form	Expanded Decimal Form	Decimal
$43\frac{65}{100}$	$40 + 3 + \frac{6}{10} + \frac{5}{100}$	$40 + 3 + 0.6 + 0.05$	43.65
$21\frac{90}{100}$			
$40\frac{76}{100}$			
$7\frac{82}{100}$			
$18\frac{3}{100}$			

4^{th} *Grade – CCSS for Mathematics*

4.NF.C.5

Standard Explanation
Fourth-grade students develop an understanding of decimal notation for fractions and compare decimal fractions (fractions with a denominator of 10 or 100). This work lays the foundation for performing operations with decimal numbers in grade five. Students learn to add decimal fractions by converting them to fractions with the same denominator (4.NF.5 ▲). For example, students express $\frac{3}{10}$ as $\frac{30}{100}$ before they add $\frac{30}{100} + \frac{4}{100} = \frac{34}{100}$. Students can use graph paper, base-ten blocks, and other place-value models to explore the relationship between fractions with denominators of 10 and 100 (adapted from UA Progressions Documents 2013a) (CA Mathematics Framework, adopted Nov. 6, 2013).

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Fraction	Expanded Fraction Form	Expanded Decimal Form	Decimal
$43\frac{65}{100}$	$40 + 3 + \frac{6}{10} + \frac{5}{100}$	$40 + 3 + 0.6 + 0.05$	43.65
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4th Grade – CCSS for Mathematics

4.NF.C Understand decimal notation for fractions, and compare decimal fractions.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Essential Skills and Concepts:

- ☐ Place value more than 0 and less than 1
- ☐ Number line
- ☐ Decimals to the right of the zero represent zeros in the denominator

Question Stems and Prompts:

- ✓ Count forward beginning at 1.
- ✓ What number comes next? How do you know?
- ✓ Count by ones.
- ✓ Count by tens.

Vocabulary	**Spanish Cognates**
Tier 2	
• more	más
• less	
• equal	igual

Standards Connections

4.NF.6 → 4.NF.7, 4.MD.2

4.NF.C Understand decimal notation for fractions, and compare decimal fractions.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Essential Skills and Concepts:

- ☐ Place value more than 0 and less than 1
- ☐ Number line
- ☐ Decimals to the right of the zero represent zeros in the denominator

Question Stems and Prompts:

- ✓ Count forward beginning at 1.
- ✓ What number comes next? How do you know?
- ✓ Count by ones.
- ✓ Count by tens.

Vocabulary	**Spanish Cognates**
Tier 2	
• more	más
• less	
• equal	igual

Standards Connections

4.NF.6 → 4.NF.7, 4.MD.2

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4.NF.B.6

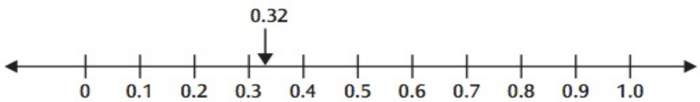
Standard Explanation

In grade four, students first use decimal notation for fractions with denominators 10 or 100 (4.NF.6 ▲), understanding that the number of digits to the right of the decimal point indicates the number of zeros in the denominator. Students make connections between fractions with denominators of 10 and 100 and place value. They read and write decimal fractions; for example, students say 0.32 as "thirty-two hundredths" and learn to flexibly write this as both 0.32 and $\frac{32}{100}$.

Focus, Coherence, and Rigor

To reinforce student understanding, teachers are urged to consistently use place-value-based language when naming decimals—for example, by saying "four-tenths" rather than "point four" when referring to 0.4, and by saying "sixty-eight hundredths" as opposed to "point sixty-eight" or "point six eight" when referring to 0.68.

Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. They reason that $\frac{32}{100}$ is a little more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$, so it would be placed on the number line near that value (MP.2, MP.4, MP.5, MP.7) (CA Mathematics Framework, adopted Nov. 6, 2013).



NF Progression Information:

Using the unit fractions $\frac{1}{10}$ and $\frac{1}{100}$, non-whole numbers like $23\frac{7}{10}$ can be written in an expanded form that extends the form used with whole numbers: $2 \times 10 + 3 \times 1 + 7 \times \frac{1}{10}$ as with whole-number expansions (Number and Operations – Fractions, 3 – 5, September 19, 2013 http://ime.math.arizona.edu/progressions/).

Illustrative Task:

- Dimes and Pennies
  http://www.illustrativemathematics.org/illustrations/152

A dime is $\frac{1}{10}$ of a dollar and a penny is $\frac{1}{100}$ of a dollar.

What fraction of a dollar is 6 dimes and 3 pennies? Write your answer in both fraction and decimal form.

IM Commentary

Students may think of this task in different ways. Some may think of the equivalence between dimes and pennies, stating that 6 dimes is equivalent to 60 pennies, thus giving a total of 63 pennies which can be represented as $\frac{63}{100}$ or 0.63 of a dollar. Others may think of $\frac{6}{10}$ as being equivalent to $\frac{60}{100}$ and then add $\frac{60}{100}$ plus $\frac{3}{100}$ to total $\frac{63}{100}$ or 0.63 of a dollar.

4th Grade – CCSS for Mathematics

4.NF.B.6

Standard Explanation

In grade four, students first use decimal notation for fractions with denominators 10 or 100 (4.NF.6 ▲), understanding that the number of digits to the right of the decimal point indicates the number of zeros in the denominator. Students make connections between fractions with denominators of 10 and 100 and place value. They read and write decimal fractions; for example, students say 0.32 as "thirty-two hundredths" and learn to flexibly write this as both 0.32 and $\frac{32}{100}$.

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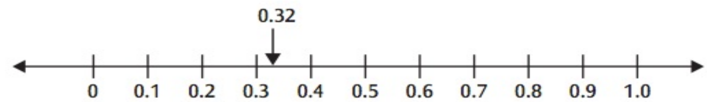
Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. They reason that $\frac{32}{100}$ is a little more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$, so it would be placed on the number line near that value (MP.2, MP.4, MP.5, MP.7) (CA Mathematics Framework, adopted Nov. 6, 2013).



NF Progression Information:

Using the unit fractions $\frac{1}{10}$ and $\frac{1}{100}$, non-whole numbers like $23\frac{7}{10}$ can be written in an expanded form that extends the form used with whole numbers: $2 \times 10 + 3 \times 1 + 7 \times \frac{1}{10}$ as with whole-number expansions (Number and Operations – Fractions, 3 – 5, September 19, 2013 http://ime.math.arizona.edu/progressions/).

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4.NF.B Understand decimal notation for fractions, and compare decimal fractions.

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using **the number line or another** visual model. **CA**

Essential Skills and Concepts:
- ☐ Number order of decimals
- ☐ Place Value
- ☐ Compare fractions with same denominator
- ☐ Same whole denominator

Question Stems and Prompts:
- ✓ ...is more than/less than …because….
- ✓ Do these fractions have a common denominator?
- ✓ Are these fractions equivalent? How do you know?

Vocabulary	**Spanish Cognates**
Tier 2	
• convert	convertir
Tier 3	
• denominator	denominador
• equivalent fractions	fracción equivalente

Standards Connections
4.NF.7 ← 4.NF.2, 4.NF.6

4.NF.7 Example:

Common Misconceptions
- Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they may think that $0.2 < 0.07$ simply because $2 < 7$.
- Students sometimes think, *"The longer the decimal number, the greater the value."* For example, they may think that 0.03 is greater than 0.3.

4[th] Grade – CCSS for Mathematics

4.NF.B Understand decimal notation for fractions, and compare decimal fractions.

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using **the number line or another** visual model. **CA**

Essential Skills and Concepts:
- ☐ Number order of decimals
- ☐ Place Value
- ☐ Compare fractions with same denominator
- ☐ Same whole denominator

Question Stems and Prompts:
- ✓ ...is more than/less than …because….
- ✓ Do these fractions have a common denominator?
- ✓ Are these fractions equivalent? How do you know?

Vocabulary / **Spanish Cognates**
Tier 2
• convert — convertir
Tier 3
• denominator — denominador
• equivalent fractions — fracción equivalente

Standards Connections
4.NF.7 ← 4.NF.2, 4.NF.6

4.NF.7 Example:

Common Misconceptions
- Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they may think that $0.2 < 0.07$ simply because $2 < 7$.
- Students sometimes think, *"The longer the decimal number, the greater the value."* For example, they may think that 0.03 is greater than 0.3.

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4.NF.B.7

Standard Explanation
Students compare two decimals to hundredths by reasoning about their size (4.NF.7 ▲). They relate their understanding of the place-value system for whole numbers to fractional parts represented as decimals. Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator and ensuring that the "wholes" are the same.

NF Progression Information:
Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator. For example, to compare 0.2 and 0.09, students think of them as 0.20 and 0.09 and see that 0.20> 0.09 because

$$\frac{20}{100} > \frac{9}{100}.$$

(Number and Operations – Fractions, 3 – 5, September 19, 2013 http://ime.math.arizona.edu/progressions/).

Illustrative Task:
- Using Place Value
  http://www.illustrativemathematics.org/illustrations/182

Task

a. Fill in the following blanks to:

0.17, **0.27**, ______, ______, ______, ______, ______, _____

______, ______, **0.56**, **0.66**, ______, ______, ______, ______

______, ______, ______, **103.12**, ______, **103.32**, ______, ______

______, ______, ______, **103.12**, ______, ______, ______, **103.16**

______, ______, ______, **103.12**, **113.12**, ______, ______, ______

- Count by tenths:
- Count by tenths:
- Count by tenths:
- Count by hundredths
- Count by tens:

b. Fill in the blank with $<$, $=$, or $>$ to make the correct comparison.

- 4 tenths + 3 hundredths _____ 2 tenths + 12 hundredths
- 3 hundredths + 4 tenths _____ 2 tenths + 22 hundredths
- 5 hundredths + 1 tenth _____ 11 tenths + 4 hundredths

4th Grade – CCSS for Mathematics

4.NF.B.7

Standard Explanation
Students compare two decimals to hundredths by reasoning about their size (4.NF.7 ▲). They relate their understanding of the place-value system for whole numbers to fractional parts represented as decimals. Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator and ensuring that the "wholes" are the same.

NF Progression Information:
Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator. For example, to compare 0.2 and 0.09, students think of them as 0.20 and 0.09 and see that 0.20> 0.09 because

$$\frac{20}{100} > \frac{9}{100}.$$

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Illustrative Task:
- Using Place Value
  http://www.illustrativemathematics.org/illustrations/182

Task

a. Fill in the following blanks to:

0.17, **0.27**, ______, ______, ______, ______, ______, _____

______, ______, **0.56**, **0.66**, ______, ______, ______, ______

______, ______, ______, **103.12**, ______, **103.32**, ______, ______

______, ______, ______, **103.12**, ______, ______, ______, **103.16**

______, ______, ______, **103.12**, **113.12**, ______, ______, ______

- Count by tenths:
- Count by tenths:
- Count by tenths:
- Count by hundredths
- Count by tens:

b. Fill in the blank with $<$, $=$, or $>$ to make the correct comparison.

- 4 tenths + 3 hundredths _____ 2 tenths + 12 hundredths
- 3 hundredths + 4 tenths _____ 2 tenths + 22 hundredths
- 5 hundredths + 1 tenth _____ 11 tenths + 4 hundredths

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4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

Essential Skills and Concepts:
- ☐ Number patterns
- ☐ Add/Multiply fluently
- ☐ Place value in the metric system
- ☐ Place value in the English system
- ☐ Two column tables

Question Stems and Prompts:
- ✓ How many meters are there in a kilometer?
- ✓ What does the prefix kilo-, hecto-, deka-, represent?
- ✓ How are Kilo-, hecto-, deca-, different from deci, centi-, and milli-.
- ✓ Describe the relationship between a gram, a meter, and a liter.

Vocabulary	**Spanish Cognates**
Tier 2	
• convert	convertir
• foot	
Tier 3	
• kilo-	kilo-
• hecto-	hecto-
• deca-	deca-
• deci-	deci-
• denti-	centi-
• milli-	mili
• yard	yarda
• mile	milla
• minute	minuto

Standards Connections
4.MD.1 →4.MD.2

4th Grade – CCSS for Mathematics

4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

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Vocabulary	**Spanish Cognates**
Tier 2	
• convert	convertir
• foot	
Tier 3	
• kilo-	kilo-
• hecto-	hecto-
• deca-	deca-
• deci-	deci-
• denti-	centi-
• milli-	mili
• yard	yarda
• mile	milla
• minute	minuto

Standards Connections
4.MD.1 →4.MD.2

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4th Grade – CCSS for Mathematics

4.MD.A.1

Standard Explanation

Students will need ample opportunities to become familiar with new units of measure. In prior years, work with units was limited to units such as pounds, ounces, grams, kilograms, and liters, and students did not convert measurements.

Students may use two-column tables to convert from larger to smaller units and record equivalent measurements. For example:

kg	g
1	1000
2	2000
3	3000

ft	in
1	12
2	24
3	36

lb	oz
1	16
2	32
3	48

(CA Mathematics Framework, adopted Nov. 6, 2013)

Geometric Measurement Progression Information:

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit Fourth graders learn the relative sizes of measurement units within a system of measurement including: *length: meter (m), kilometer (km), centimeter (cm), millimeter (mm); volume: liter (l), milliliter (ml, 1 cubic centimeter of water; a liter, then, is 1000 ml); mass: gram (g, about the weight of a cc of water), kilogram (kg); time: hour (hr), minute (min), second (sec)* (Geometric Measurement, K – 5, June 23, 2012 http://ime.math.arizona.edu/progressions/).

Illustrative Task:

- Who is the tallest? https://www.illustrativemathematics.org/content-standards/4/MD/A/1/tasks/1508

Mr. Liu asked the students in his fourth grade class to measure their heights. Here are some of the heights they recorded:

Student	Height
Sarah	50 inches
Jake	$4\frac{1}{4}$ feet
Andy	$1\frac{1}{2}$ yards
Emily	4 feet and 4 inches

List the four students from tallest to shortest.

4.MD.A.1

Standard Explanation

Students will need ample opportunities to become familiar with new units of measure. In prior years, work with units was limited to units such as pounds, ounces, grams, kilograms, and liters, and students did not convert measurements.

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4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Essential Skills and Concepts:
- ☐ Add/Subtract fluently
- ☐ Multiply/Divide fluently
- ☐ Use a number line correctly

Question Stems and Prompts:
- ✓ How much time has passed?
- ✓ If you paid with a $20 bill, how much change would you receive?
- ✓ If you ran 2 miles, how many feet did you run?

Vocabulary	**Spanish Cognates**
Tier 2	
• money	
• mass	masa
• convert	convertir
• distance	distancia
Tier 3	
• number line	línea numerica
• volume	volumen
• liters	litro
• meter(s)	metro(s)
• mile(s)	milla(s)
• gallon(s)	galones

Standards Connections
4.MD.2 → 4.OA.3, 4.NF.3d

Focus, Coherence, and Rigor

In grade four, students use the four operations to solve word problems involving measurement quantities such as liquid volume, mass, and time (**4.MD.1–2**). Measurement provides a context for solving problems using the four operations and connects to and supports major grade-level work in the cluster "Use the four operations with whole numbers to solve problems" (**4.OA.1–3▲**) and clusters in the domain "Number and Operations—Fractions" (**4.NF.1–4▲**). For example, students use whole-number multiplication to express measurements given in a larger unit in terms of a smaller unit, and they solve word problems involving addition and subtraction of fractions or multiplication of a fraction by a whole number.

Adapted from PARCC 2012.

(CA Mathematics Framework, adopted Nov. 6, 2013)

4th Grade – CCSS for Mathematics

4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Essential Skills and Concepts:
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Tier 3	
• number line	línea numerica
• volume	volumen
• liters	litro
• meter(s)	metro(s)
• mile(s)	milla(s)
• gallon(s)	galones

Standards Connections
4.MD.2 → 4.OA.3, 4.NF.3d

Focus, Coherence, and Rigor

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4th Grade – CCSS for Mathematics

4.MD.A.2

Standard Explanation
Students in grade four begin using the four operations to solve word problems involving measurement quantities such as liquid volume, mass, and time (4.MD.2), including problems involving simple fractions or decimals.

Examples: Word Problems Involving Measures	4.MD.2
1. **Division/Fractions:** Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get? Students may record their solutions by using fractions or inches. ***Solution:*** The answer would be $\frac{2}{3}$ of a foot, or 8 inches. Students are able to express the answer in inches because they understand that $\frac{1}{3}$ of a foot is 4 inches and $\frac{2}{3}$ of a foot is 2 groups of $\frac{1}{3}$.	
2. **Addition:** Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran? ***Solution:*** Students know that 60 minutes make up one hour. We know Mason ran one hour, which is 60 minutes. He also ran 15 + 25 + 40 = 80 minutes more, which makes 140 total minutes.	
3. **Multiplication:** Mario and his two brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have? ***Solution:*** Students know that 1 liter is 1000 milliliters (ml), so Mario brought 1000 + 500 = 1500 ml, and Javier brought 2 × 1000 = 2000 ml. This means the three brothers had a total of 1500 + 2000 + 450 = 3950 ml.	
Adapted from ADE 2010.	

(CA Mathematics Framework, adopted Nov. 6, 2013)

Geometric Measurement Progression Information:
Students also combine competencies from different domains as they solve measurement problems using all four arithmetic operations, addition, subtraction, multiplication, and division. For example, "How many liters of juice does the class need to have at least 35 cups if each cup takes 225 ml?" Students may use tape or number line diagrams for solving such problems (MP1) (Geometric Measurement, K – 5, June 23, 2012 http://ime.math.arizona.edu/progressions/).

Illustrative Task:
- Margie Buys Apples

https://www.illustrativemathematics.org/illustrations/873

Task

Margie bought 3 apples that cost 50 cents each. She paid with a five-dollar bill. How much change did Margie receive?

4.MD.A.2

Standard Explanation
Students in grade four begin using the four operations to solve word problems involving measurement quantities such as liquid volume, mass, and time (4.MD.2), including problems involving simple fractions or decimals.

Examples: Word Problems Involving Measures	4.MD.2
1. **Division/Fractions:** Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get? Students may record their solutions by using fractions or inches. ***Solution:*** The answer would be $\frac{2}{3}$ of a foot, or 8 inches. Students are able to express the answer in inches because they understand that $\frac{1}{3}$ of a foot is 4 inches and $\frac{2}{3}$ of a foot is 2 groups of $\frac{1}{3}$.	
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4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Essential Skills and Concepts:
- ☐ Multiplying fluently
- ☐ Dividing fluently
- ☐ Variables

Question Stems and Prompts:
- ✓ Draw/model the situation.
- ✓ How is perimeter different from area?
- ✓ What is the relationship between area and perimeter?

Vocabulary	**Spanish Cognates**
• area	área
• perimeter	perímetro

Standards Connections
4.MD.3 ← 3.OA.4, 3.MD.7b, 3.MD.8

4.MD.3 Examples:

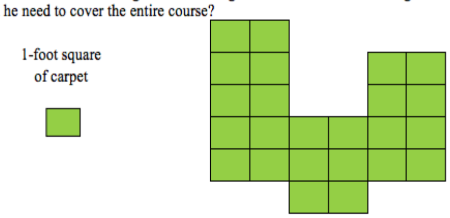
Example: Area and Perimeter of Rectangles (MP.2, MP.4) 4.MD.3

Sally wants to build a pen for her dog, Callie. Her parents give her $200 to buy the fencing material, but they want Sally to design the pen. Her parents suggest that she consider different plans. Her parents also remind her that Callie needs as much room as possible to run and play, that the pen can be placed anywhere in the yard, and that the wall of the house could be used as one side of the pen. Sally decides to buy fencing material that costs $8.50 per foot. She also needs at least one three-foot-wide gate for the pen that costs $15.

- Design a pen for Callie. Experiment with different pen designs and consider the advice from Sally's parents. Sally's house can be any configuration.
- Write a letter to Sally with your diagrams and calculations. Explain why some designs are better for Callie than others.

(CA Mathematics Framework, adopted Nov. 6, 2013)

Ar. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet vill he need to cover the entire course?



(Adapted from NC Public Schools)

4th Grade – CCSS for Mathematics

4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

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Question Stems and Prompts:
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- area — área
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Standards Connections
4.MD.3 ← 3.OA.4, 3.MD.7b, 3.MD.8

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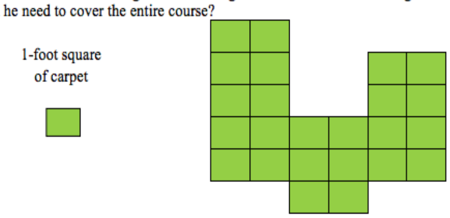

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4.MD.A Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Standard Explanation
In grade three, students developed an understanding of area and perimeter by using visual models. Students in grade four are expected to use formulas to calculate area and perimeter of rectangles; however, they still need to understand and be able to communicate their understanding of why the formulas work. It is still important for students to draw length units or square units inside a small rectangle to keep the distinction fresh and visual, and some students may still need to write the lengths of all four sides before finding the perimeter. Students know that answers for the area formula ($\ell \times w$) will be in square units and that answers for the perimeter formula ($2\ell \times 2w$ or $2(\ell \times w)$) will be in linear units (adapted from ADE 2010) (CA Mathematics Framework, adopted Nov. 6, 2013).

Measurement Data Progression Information:
Students learn to apply these understandings and formulas to
the solution of real-world and mathematical problems.4.MD.3 For ex-ample, they might be asked, "A rectangular garden has as an area of 80 square feet. It is 5 feet wide. How long is the garden?" Here, specifying the area and the width, creates an unknown factor problem (see Table 1). Similarly, students could solve perimeter problems that give the perimeter and the length of one side and ask the length of the adjacent side. Students could be challenged to solve multistep problems such as the following. "A plan for a house includes rectangular room with an area of 60 square meters and a perimeter of 32 meters. What are the length and the width of the room?" (Geometric Measurement, K – 5, June 23, 2012 http://ime.math.arizona.edu/progressions/).

Illustrative Task(s):
- Karl's Garden https://www.illustrativemathematics.org/illustrations/876

Task

Karl's rectangular vegetable garden is 20 feet by 45 feet, and Makenna's is 25 feet by 40 feet. Whose garden is larger in area?

4th Grade – CCSS for Mathematics

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4th Grade – CCSS for Mathematics

4.MD.B Represent and interpret data

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Essential Skills and Concepts:

- ☐ Count fluently
- ☐ Plot units on a number line
- ☐ Converting fractions
- ☐ Add/subtract fractions

Question Stems and Prompts:

- ✓ Place the following fractions, …. in order on a number line
- ✓ Are these fractions equal?
- ✓ Which fraction is larger/smaller?
- ✓ Justify that one is larger/smaller.
- ✓ Explain why you put the fractions in that order.

Vocabulary	**Spanish Cognates**
Tier 3	
• line plot	
• specimens	espécimen
• data	datos

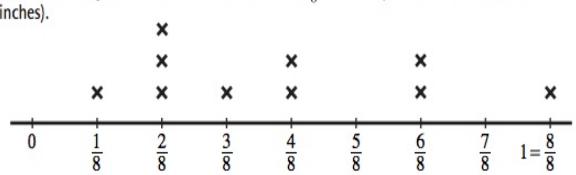
Standards Connections

4.MD.4 → 4.NF.3d

4.MD.4 Example:

Example: Interpreting Line Plots 4.MD.4

Ten students measure objects in their desk to the nearest $\frac{1}{8}$ inch. They record their results on the line plot below (in inches).



Possible related questions:

- How many objects measured $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ inch?
- If you put the objects end to end, what would the total length be?
- If five $\frac{1}{8}$-inch pencils are placed end to end, what would the total length of the pencils be?

Adapted from ADE 2010.

(CA Mathematics Framework, adopted Nov. 6, 2013)

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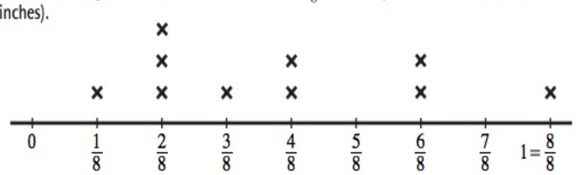

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(CA Mathematics Framework, adopted Nov. 6, 2013)

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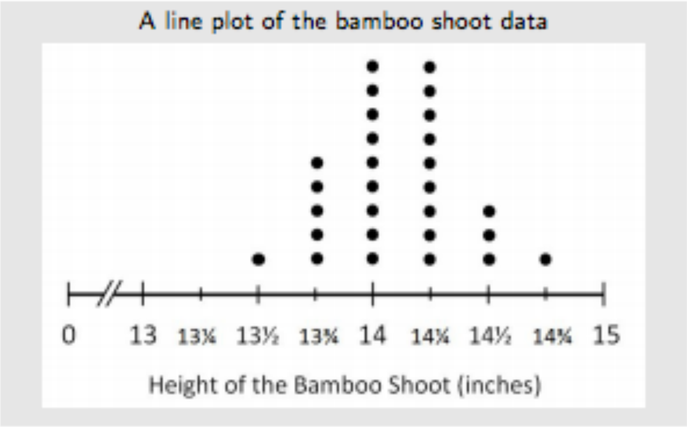
Standard Explanation

As students work with data in kindergarten through grade five, they build foundations for the study of statistics and probability in grades six and beyond, and they strengthen and apply what they learn in arithmetic. Fourth-grade students make a line plot to display a data set of measurements in fractions of a unit $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8})$ and they solve problems involving addition and subtraction of fractions by using information presented in line plots (4.MD.4) (CA Mathematics Framework, adopted Nov. 6, 2013).

Measurement Data Progression Information:

Students learn elements of fraction equivalence and arithmetic, including multiplying a fraction by a whole number and adding and subtracting fractions with like denominators. Students can use these skills to solve problems, including problems that arise from analyzing line plots. For example, with reference to the line plot below, students might find the difference between the greatest and least values in the values in the data. (In solving such problems, students may need to label the measurement scale in eights so as to produce like denominators. Decimal data can also be used in this grade.)



(K-3, Categorical Data; Grades 2-5, Measurement Data, June 20, 2011 http://ime.math.arizona.edu/progressions/).

4th Grade – CCSS for Mathematics

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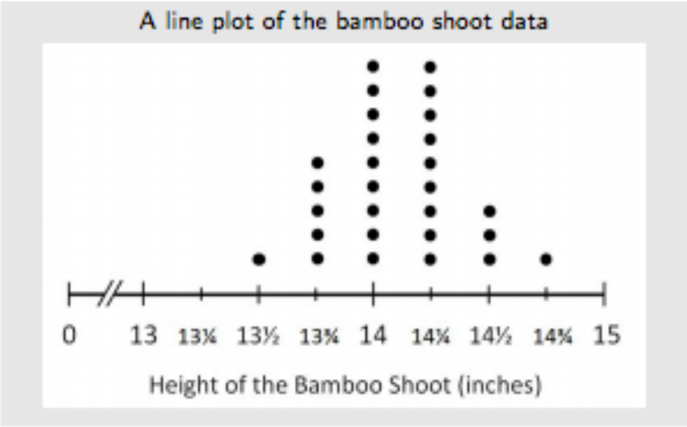

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4.MD.C Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

Essential Skills and Concepts:
- ☐ Find angle measures
- ☐ Use a protractor
- ☐ Find the center of a circle

Question Stems and Prompts:
- ✓ Count forward beginning at 1.
- ✓ What number comes next? How do you know?
- ✓ Count by ones.
- ✓ Count by tens.

Vocabulary	**Spanish Cognates**
Tier 2	
• intersect	
• ray	rayo
Tier 3	
• angle	ángulo
• circular arc	arco circular
• degrees	

Standards Connections
4.MD.5 → 4.MD.6, 4.MD.7, 4.G.2

4.MD.5 Example:

An angle

name	measurement
right angle	90°
straight angle	180°
acute angle	between 0 and 90°
obtuse angle	between 90° and 180°
reflex angle	between 180° and 360°

(Geometric Measurement, K – 5, June 23, 2012 http://ime.math.arizona.edu/progressions/)

4th Grade – CCSS for Mathematics

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Tier 2	
• intersect	
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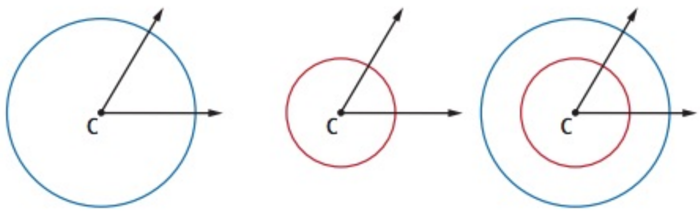
4th Grade – CCSS for Mathematics

4.MD.C.5

Standard Explanation

Students in grade four learn that angles are geometric shapes formed by two rays that share a common endpoint (4.MD.5). They understand angle measure as being that portion of a circular arc that is formed by the angle when a circle is centered at their shared vertex. The following figure helps students see that an angle is determined by the arc it creates relative to the size of the entire circle, evidenced by the picture showing two angles of the same measure (though their circles are not the same).

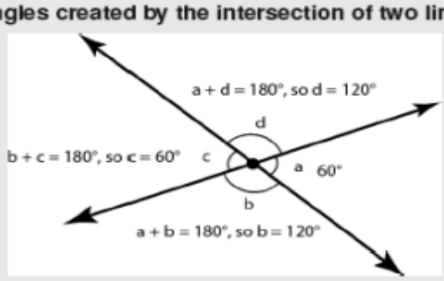


However, the pie-shaped pieces formed by each angle are not the same size; this shows that angle measure is not defined in terms of these areas. The angle in each case is 60°, since it measures an arc that is the total circumference of the circle in both the larger and smaller circles—but the pie-shaped pieces formed by the angle have different areas (CA Mathematics Framework, adopted Nov. 6, 2013).

Focus, Coherence, and Rigor

Students' work with concepts of angle measures (**4.MD.5a, 4.MD.7**) also connects to and supports the addition of fractions, which is major work at the grade in the cluster "Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers" (**4.NF.3–4▲**). For example, a 1° measure is a fraction of an entire rotation, and adding angle measures together is the same as adding fractions with a denominator of 360.

Angles created by the intersection of two lines



When two lines intersect, they form four angles. If the measurement of one is known (e.g., angle a is 60°), the measurement of the other three can be determined.

(Geometric Measurement, K – 5, June 23, 2012 http://ime.math.arizona.edu/progressions/)

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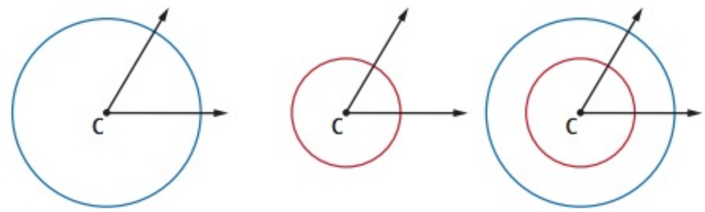

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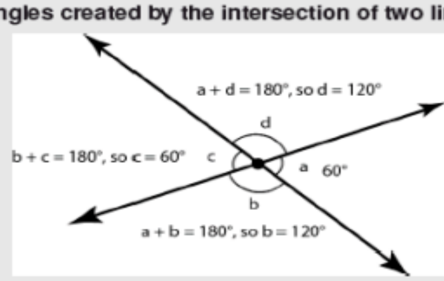

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4.MD.C Geometric measurement: understand concepts of angle and measure angles.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

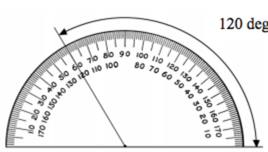
Essential Skills and Concepts:
- ☐ Use a protractor correctly
- ☐ Sketch angles using a protractor

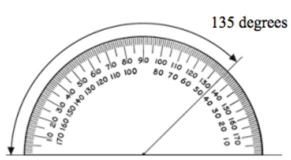
Question Stems and Prompts:
- ✓ Sketch a(n) ____ degree angle
- ✓ Sketch an angle that is larger than 90 degrees.
- ✓ Sketch an angle that is less than 90 degrees.
- ✓ Sketch an angle that is more than 90 degrees and less than 180 degrees.

Vocabulary	Spanish Cognates
Tier 3	
• degrees	
• protractor	
• angle	ángulo
• right angle	
• straight line	
• obtuse angle	ángulo obtuso
• acute angle	ángulo agudo

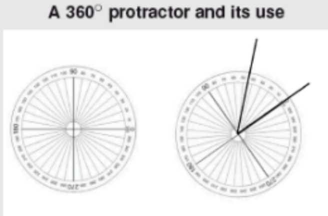
Standards Connections
4.MD.6 ← 4.MD.5

4.MD.5 Example:





A 360° protractor and its use



The figure on the right shows a protractor being used to measure a 45° angle. The protractor is placed so that one side of the angle lies on the line corresponding to 0° on the protractor and the other side of the angle is located by a clockwise rotation from that line.

(Geometric Measurement, K – 5, June 23, 2012 http://ime.math.arizona.edu/progressions/)

4th Grade – CCSS for Mathematics

4.MD.C Geometric measurement: understand concepts of angle and measure angles.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Essential Skills and Concepts:
- ☐ Use a protractor correctly
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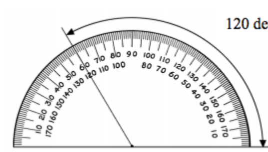
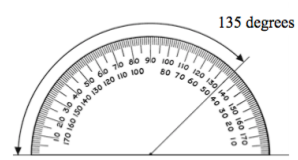
Question Stems and Prompts:
- ✓ Sketch a(n) ____ degree angle
- ✓ Sketch an angle that is larger than 90 degrees.
- ✓ Sketch an angle that is less than 90 degrees.
- ✓ Sketch an angle that is more than 90 degrees and less than 180 degrees.

Vocabulary / Spanish Cognates
Tier 3
• degrees
• protractor
• angle — ángulo
• right angle
• straight line
• obtuse angle — ángulo obtuso
• acute angle — ángulo agudo

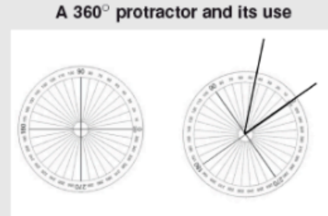
Standards Connections
4.MD.6 ← 4.MD.5

4.MD.5 Example:





A 360° protractor and its use



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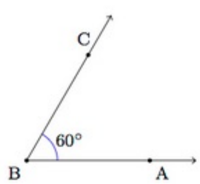
4.MD.C Geometric measurement: understand concepts of angle and measure angles.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Standard Explanation
Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular). Students recognize angle measure as additive and use this to solve addition and subtraction problems to find unknown angles on a diagram (CA Mathematics Framework, adopted Nov. 6, 2013).

Illustrative Task:
- Measuring Angles https://www.illustrativemathematics.org/illustrations/909

a. Draw an angle that measures 60 degrees like the one shown here:



b. Draw another angle that measures 25 degrees. It should have the same vertex and share side $\overrightarrow{BA}$.

c. How many angles are there in the figure you drew? What are their measures?

d. Make a copy of your 60 degree angle. Draw a different angle that measures 25 degrees and has the same vertex and also shares side $\overrightarrow{BA}$.

e. How many angles are there in the figure you drew? What are their measures?

4th Grade – CCSS for Mathematics

4.MD.C Geometric measurement: understand concepts of angle and measure angles.

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Standard Explanation
Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular). Students recognize angle measure as additive and use this to solve addition and subtraction problems to find unknown angles on a diagram (CA Mathematics Framework, adopted Nov. 6, 2013).

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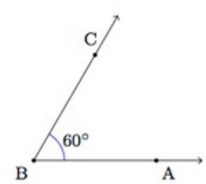

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4th Grade – CCSS for Mathematics

4.MD.C Geometric measurement: understand concepts of angle and measure angles.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

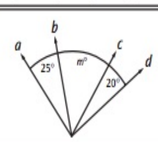
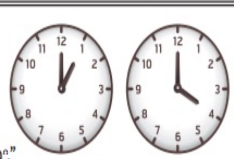
Essential Skills and Concepts:
- ☐ Use a protractor correctly
- ☐ Add/Subtract fluently
- ☐ Solve for a missing angle

Question Stems and Prompts:
- ✓ Think of a way to find the missing angle?
- ✓ Why do you think that will work?
- ✓ Can you prove that your idea will work every time?
- ✓ Create another problem and prove to me that it will work again.

Vocabulary	**Spanish Cognates**
Tier 2	
• missing angle	
• non-overlapping	
• decomposed	descompuesto
Tier 3	
• variable	variable
• protractor	
• additive	aditivo

Standards Connections
4.MD.7 ← 4.MD.5

4.MD.7 Examples:

Examples: Angle measure is additive.	4.MD.7 (MP.1, MP.2, MP.4, MP.7)
1. If ray *a* is perpendicular to ray *d* (see 4.G.1), what is the value of *m*? *Solution:* "Since perpendicular lines make an angle that measures 90°, I know that $25 + m + 20 = 90$. This means that $m = 90 - 45 = 45$."	
2. Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30°. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4? *Solution:* "This looks like it is four times as much, so it is $4 \times 30° = 120°$." Adapted from ADE 2010.	

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4.MD.C Geometric measurement: understand concepts of angle and measure angles.

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Tier 2	
• missing angle	
• non-overlapping	
• decomposed	descompuesto
Tier 3	
• variable	variable
• protractor	
• additive	aditivo

Standards Connections
4.MD.7 ← 4.MD.5

4.MD.7 Examples:

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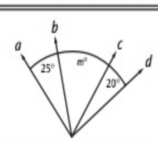
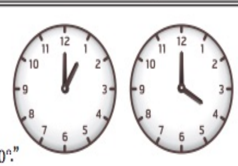
1. If ray *a* is perpendicular to ray *d* (see 4.G.1), what is the value of *m*? *Solution:* "Since perpendicular lines make an angle that measures 90°, I know that $25 + m + 20 = 90$. This means that $m = 90 - 45 = 45$."	
2. Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30°. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4? *Solution:* "This looks like it is four times as much, so it is $4 \times 30° = 120°$." Adapted from ADE 2010.	

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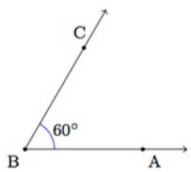
4.MD.C.7

Standard Explanation
Before students solve word problems involving unknown angle measures (4.MD.7), they need to understand concepts of angle measure (4.MD.5) and, presumably, gain some experience measuring angles (4.MD.6). Students also need some familiarity with the geometric terms that are used to define angles as geometric shapes (4.G.1) [adapted from PARCC 2012] (CA Mathematics Framework, adopted Nov. 6, 2013).

Illustrative Tasks:

- Measuring Angles
  https://www.illustrativemathematics.org/illustrations/909

a. Draw an angle that measures 60 degrees like the one shown here:



b. Draw another angle that measures 25 degrees. It should have the same vertex and share side $\overrightarrow{BA}$.

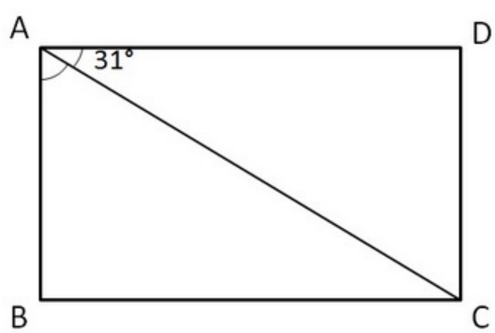
c. How many angles are there in the figure you drew? What are their measures?

d. Make a copy of your 60 degree angle. Draw a different angle that measures 25 degrees and has the same vertex and also shares side $\overrightarrow{BA}$.

e. How many angles are there in the figure you drew? What are their measures?

- Finding an Unknown Angle
  https://www.illustrativemathematics.org/content-standards/4/MD/C/7/tasks/1168

In the figure, $ABCD$ is a rectangle and $\angle CAD = 31^\circ$. Find $\angle BAC$.



4[th] Grade – CCSS for Mathematics

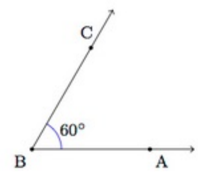
4.MD.C.7

Standard Explanation
Before students solve word problems involving unknown angle measures (4.MD.7), they need to understand concepts of angle measure (4.MD.5) and, presumably, gain some experience measuring angles (4.MD.6). Students also need some familiarity with the geometric terms that are used to define angles as geometric shapes (4.G.1) [adapted from PARCC 2012] (CA Mathematics Framework, adopted Nov. 6, 2013).

Illustrative Tasks:

- Measuring Angles
  https://www.illustrativemathematics.org/illustrations/909

a. Draw an angle that measures 60 degrees like the one shown here:



b. Draw another angle that measures 25 degrees. It should have the same vertex and share side $\overrightarrow{BA}$.

c. How many angles are there in the figure you drew? What are their measures?

d. Make a copy of your 60 degree angle. Draw a different angle that measures 25 degrees and has the same vertex and also shares side $\overrightarrow{BA}$.

e. How many angles are there in the figure you drew? What are their measures?

- Finding an Unknown Angle
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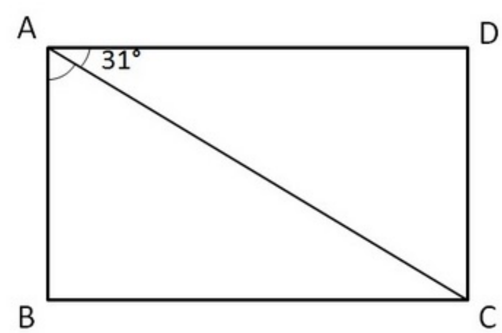
In the figure, $ABCD$ is a rectangle and $\angle CAD = 31^\circ$. Find $\angle BAC$.



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4th Grade – CCSS for Mathematics

4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

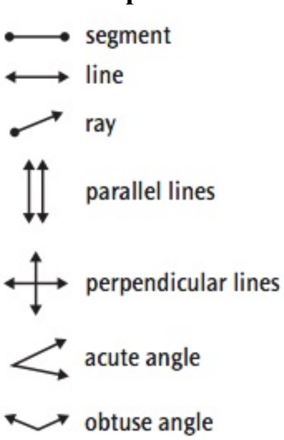
Essential Skills and Concepts:
- ☐ Use a protractor correctly
- ☐ Sketch geometrical figures

Question Stems and Prompts:
- ✓ Verbally describe what _____ looks like.
- ✓ Sketch what _____ should look like.
- ✓ Teacher sketches an incorrect example and has a student explain why it is an incorrect example.
- ✓ What would be the correct name for this figure? (Teacher shows/draws various geometric figures)

Vocabulary	**Spanish Cognates**
Tier 2	
• points	puntos
• lines	línea
• rays	rayo
Tier 3	
• two-dimensional	
• line segment	segmento de línea
• angles	ángulo
• right angle	
• acute angle	ángulo agudo
• obtuse angle	ángulo obtuso
• perpendicular lines	perpendiculares
• parallel lines	líneas paralelas

Standards Connections
4.G.1 → 4.G.2, 4.MD.5

4.G.1 Examples:



(CA Mathematics Framework, adopted Nov. 6, 2013)

4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

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- ✓ Verbally describe what _____ looks like.
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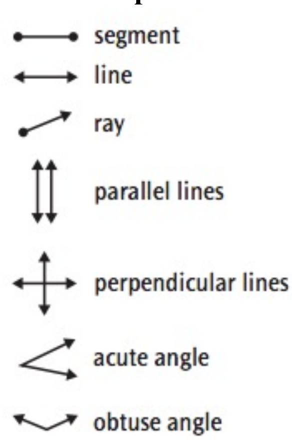
- ✓ What would be the correct name for this figure? (Teacher shows/draws various geometric figures)

Vocabulary	**Spanish Cognates**
Tier 2	
• points	puntos
• lines	línea
• rays	rayo
Tier 3	
• two-dimensional	
• line segment	segmento de línea
• angles	ángulo
• right angle	
• acute angle	ángulo agudo
• obtuse angle	ángulo obtuso
• perpendicular lines	perpendiculares
• parallel lines	líneas paralelas

Standards Connections
4.G.1 → 4.G.2, 4.MD.5

4.G.1 Examples:



(CA Mathematics Framework, adopted Nov. 6, 2013)

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4.G.A.1

Standard Explanation

A critical area of instruction in grade four is for students to understand that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

In grade four, students are exposed to the concepts of rays, angles, and perpendicular and parallel lines (4.G.1) for the first time. In addition, students classify figures based on the presence and absence of parallel or perpendicular lines and angles (4.G.2). It is helpful to provide students with a visual reminder of examples of points, line segments, lines, angles, parallelism, and perpendicularity. For example, a wall chart with the images shown at right could be displayed in the classroom. Students need to see all of these representations in different orientations. Students could draw these in different orientations and decide if all of the drawings are correct. They also need to see and draw the range of angles that are acute and obtuse. Two-dimensional figures may be classified according to characteristics, such as the presence of parallel or perpendicular lines or by angle measurements. Students may use transparencies with lines drawn on them to arrange two lines in different ways to determine that the two lines might intersect at one point or might never intersect, thereby understanding the notion of parallel lines. Further investigations may be initiated with geometry software. These types of explorations may lead to a discussion on angles.

Students' prior experience with drawing and identifying right, acute, and obtuse angles helps them classify two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles. Right triangles (triangles with one right angle) can be a category for classification, with subcategories—for example, an isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides (CA Mathematics Framework, adopted Nov. 6, 2013).

Illustrative Tasks:

- The Geometry of Letters
  https://www.illustrativemathematics.org/content-standards/4/G/A/1/tasks/1263

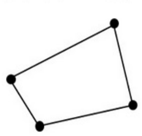
Letters can be thought of as geometric figures.

A B C D E F G
H I J K L M N
O P Q R S T U
V W X Y Z

- What's the Point?
  https://www.illustrativemathematics.org/content-standards/4/G/A/1/tasks/1272

The students in Ms. Sun's class were drawing geometric figures. First she asked them to draw some points, and then she asked them to draw all the line segments they could that join two of their points.

a. Joni drew 4 points and then drew 4 line segments between them:



4th Grade – CCSS for Mathematics

4.G.A.1

Standard Explanation

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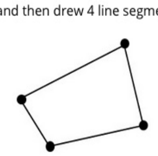

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4th Grade – CCSS for Mathematics

4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **(Two dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA**

Essential Skills and Concepts:
- ☐ Identify and name geometrical figures
- ☐ State the rules of different geometrical figures
- ☐ Recognize/categorize different triangles based on their side length
- ☐ Recognize/categorize different quadrilaterals based on their side length and angle measure

Question Stems and Prompts:
- ✓ Identify this geometrical figure and justify your reasoning.
- ✓ What is the similarity/difference between ….and ….
- ✓ Find a pattern between...and …

Vocabulary	**Spanish Cognates**
Tier 2	
• points	puntos
• lines	línea
• rays	rayo
Tier 3	
• two-dimensional	
• line segment	segmento de línea
• angles	ángulo
• right angle	
• acute angle	ángulo agudo
• obtuse angle	ángulo obtuso
• perpendicular lines	perpendiculares
• parallel lines	líneas paralelas
• equilateral triangle	triángulo equilátero
• isosceles triangle	triángulo isósceles
• scalene triangle	triángulo escaleno
• quadrilaterals	cuadrilátero
• rhombus	rombo
• square	
• rectangle	rectángulo
• parallelogram	paralelogramo
• trapezoid	trapezoide

Standards Connections
4.G.2 ← 4.G.1, 4.MD.5

4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **(Two dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA**

Essential Skills and Concepts:
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- ☐ Recognize/categorize different quadrilaterals based on their side length and angle measure

Question Stems and Prompts:
- ✓ Identify this geometrical figure and justify your reasoning.
- ✓ What is the similarity/difference between ….and ….
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Vocabulary	**Spanish Cognates**
Tier 2	
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Tier 3	
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• perpendicular lines	perpendiculares
• parallel lines	líneas paralelas
• equilateral triangle	triángulo equilátero
• isosceles triangle	triángulo isósceles
• scalene triangle	triángulo escaleno
• quadrilaterals	cuadrilátero
• rhombus	rombo
• square	
• rectangle	rectángulo
• parallelogram	paralelogramo
• trapezoid	trapezoide

Standards Connections
4.G.2 ← 4.G.1, 4.MD.5

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4th Grade – CCSS for Mathematics

4.G.A.2

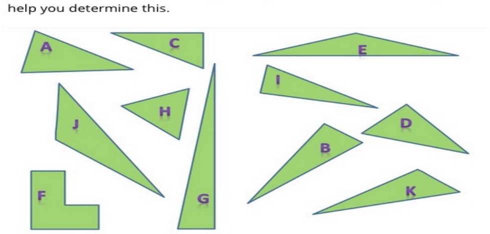
Standard Explanation
A critical area of instruction in grade four is for students to understand that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

In grade four, students are exposed to the concepts of rays, angles, and perpendicular and parallel lines (4.G.1) for the first time. In addition, students classify figures based on the presence and absence of parallel or perpendicular lines and angles (4.G.2). It is helpful to provide students with a visual reminder of examples of points, line segments, lines, angles, parallelism, and perpendicularity. For example, a wall chart with the images shown at right could be displayed in the classroom. Students need to see all of these representations in different orientations. Students could draw these in different orientations and decide if all of the drawings are correct. They also need to see and draw the range of angles that are acute and obtuse. Two-dimensional figures may be classified according to characteristics, such as the presence of parallel or perpendicular lines or by angle measurements. Students may use transparencies with lines drawn on them to arrange two lines in different ways to determine that the two lines might intersect at one point or might never intersect, thereby understanding the notion of parallel lines. Further investigations may be initiated with geometry software. These types of explorations may lead to a discussion on angles.

Illustrative Tasks:

- Are these right?
  https://www.illustrativemathematics.org/content-standards/4/G/A/2/tasks/1273

a. Which of the polygons are right triangles? Choose a measuring tool to help you determine this.



- Defining Attributes of Rectangles and Parallelograms
  https://www.illustrativemathematics.org/content-standards/4/G/A/2/tasks/1275

a. Look at each figure. Read each of the descriptions. Place an X in the box if it appears to describe the figure pictured.

	A.	B.	C.	D.
4 vertices				
Four sides				
Opposite sides parallel				
Perpendicular sides				

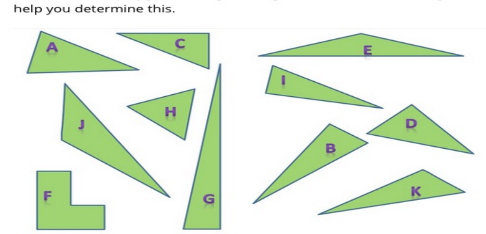
4.G.A.2

Standard Explanation
A critical area of instruction in grade four is for students to understand that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

In grade four, students are exposed to the concepts of rays, angles, and perpendicular and parallel lines (4.G.1) for the first time. In addition, students classify figures based on the presence and absence of parallel or perpendicular lines and angles (4.G.2). It is helpful to provide students with a visual reminder of examples of points, line segments, lines, angles, parallelism, and perpendicularity. For example, a wall chart with the images shown at right could be displayed in the classroom. Students need to see all of these representations in different orientations. Students could draw these in different orientations and decide if all of the drawings are correct. They also need to see and draw the range of angles that are acute and obtuse. Two-dimensional figures may be classified according to characteristics, such as the presence of parallel or perpendicular lines or by angle measurements. Students may use transparencies with lines drawn on them to arrange two lines in different ways to determine that the two lines might intersect at one point or might never intersect, thereby understanding the notion of parallel lines. Further investigations may be initiated with geometry software. These types of explorations may lead to a discussion on angles.

Illustrative Tasks:
- Are these right?
  https://www.illustrativemathematics.org/content-standards/4/G/A/2/tasks/1273

a. Which of the polygons are right triangles? Choose a measuring tool to help you determine this.



- Defining Attributes of Rectangles and Parallelograms
  https://www.illustrativemathematics.org/content-standards/4/G/A/2/tasks/1275

a. Look at each figure. Read each of the descriptions. Place an X in the box if it appears to describe the figure pictured.

	A.	B.	C.	D.
4 vertices				
Four sides				
Opposite sides parallel				
Perpendicular sides				

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4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Essential Skills and Concepts:
- ☐ Recognize a line of symmetry
- ☐ Sketch lines of symmetry

Question Stems and Prompts:
- ✓ How can we fold this shape so that both sides are identical?

Vocabulary	**Spanish Cognates**
Tier 3	
• symmetry	simetría

Standards Connections
4.G.3 ← 1.G.2

4th Grade – CCSS for Mathematics

4.G.A Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Essential Skills and Concepts:
- ☐ Recognize a line of symmetry
- ☐ Sketch lines of symmetry

Question Stems and Prompts:
- ✓ How can we fold this shape so that both sides are identical?

Vocabulary **Spanish Cognates**
Tier 3
• symmetry simetría

Standards Connections
4.G.3 ← 1.G.2

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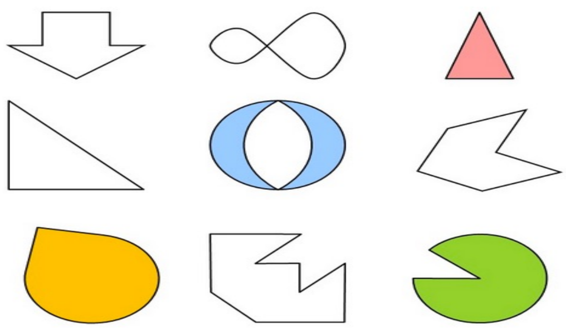
4.G.A.3

Standard Explanation
Finally, students recognize a line of symmetry for a two-dimensional figure as a line across the figure, such that the figure can be folded along the line into matching parts (adapted from ADE 2010). (CA Mathematics Framework, adopted Nov. 6, 2013)

Progression Information:
Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry (K-6, Geometry, December 27, 2014 http://ime.math.arizona.edu/progressions/).

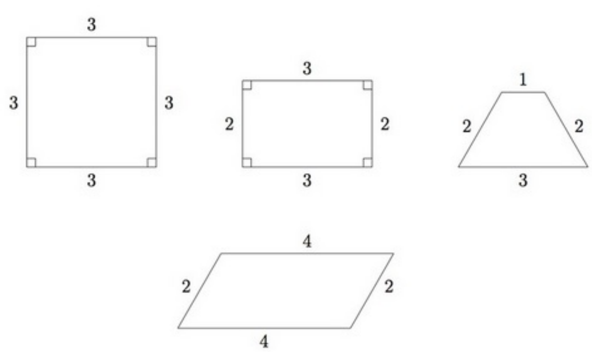
Illustrative Tasks:
- Finding Lines of Symmetry
  https://www.illustrativemathematics.org/content-standards/4/G/A/3/tasks/676

a. Each shape below has a line of symmetry. Draw a line of symmetry for each shape.



- Lines of Symmetry for Quadrilaterals
  https://www.illustrativemathematics.org/content-standards/4/G/A/3/tasks/1059

Below are pictures of four quadrilaterals: a square, a rectangle, a trapezoid and a parallelogram.



For each quadrilateral, find and draw all lines of symmetry.

4th Grade – CCSS for Mathematics

4.G.A.3

Standard Explanation
Finally, students recognize a line of symmetry for a two-dimensional figure as a line across the figure, such that the figure can be folded along the line into matching parts (adapted from ADE 2010). (CA Mathematics Framework, adopted Nov. 6, 2013)

Progression Information:
Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry (K-6, Geometry, December 27, 2014 http://ime.math.arizona.edu/progressions/).

Illustrative Tasks:
- Finding Lines of Symmetry
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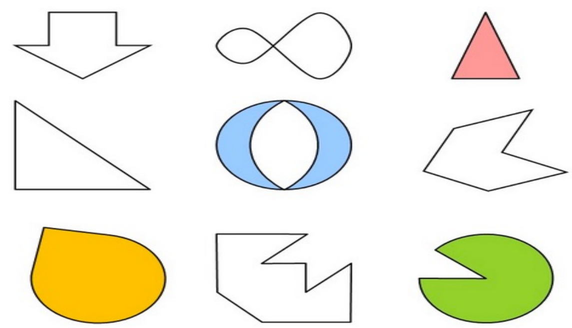
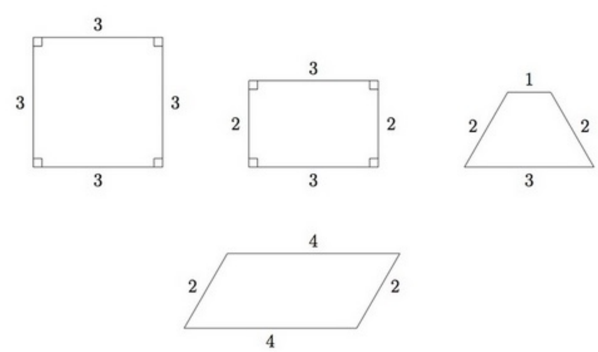
a. Each shape below has a line of symmetry. Draw a line of symmetry for each shape.



- Lines of Symmetry for Quadrilaterals
  https://www.illustrativemathematics.org/content-standards/4/G/A/3/tasks/1059

Below are pictures of four quadrilaterals: a square, a rectangle, a trapezoid and a parallelogram.



For each quadrilateral, find and draw all lines of symmetry.

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Resources for the CCSS 4th Grade Bookmarks

California *Mathematics Framework,* adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

Common Core Standards Writing Team. Progressions for the Common Core State Standards in Mathematics Tucson, AZ: Institute for Mathematics and Education, University of Arizona (Drafts), http://ime.math.arizona.edu/progressions/

- K, Counting and Cardinality; K – 5 Operations and Algebraic Thinking (2011, May 29)
- K – 5, Number and Operations in Base Ten (2012, April 21)
- K – 3, Categorical Data; Grades 2 – 5, Measurement Data* (2011, June 20)
- K – 5, Geometric Measurement (2012, June 23)
- K – 6, Geometry (2012, June 23)
- Number and Operations – Fractions, 3 – 5 (2013, September 19)

Illustrative Mathematics™ was originally developed at the University of Arizona (2011), nonprofit corporation (2013), Illustrative Tasks, http://www.illustrativemathematics.org/

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

North Carolina Department of Public Instruction, Instructional Support Tools for Achieving New Standards, Math Unpacking Standards 2012, http://www.ncpublicschools.org/acre/standards/common-core-tools/ - unmath

Common Core Flipbooks 2012, Kansas Association of Teachers of Mathematics (KATM) http://www.katm.org/baker/pages/common-core-resources.php

4th Grade – CCSS for Mathematics

Resources for the CCSS 4th Grade Bookmarks

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Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

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Illustrative Mathematics™ was originally developed at the University of Arizona (2011), nonprofit corporation (2013), Illustrative Tasks, http://www.illustrativemathematics.org/

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

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