Supporting California's Standards

Mathematics Bookmarks

Standards Reference to Support Planning and Instruction

6th Grade

Tulare County Office of Education

Tim A. Hire, County Superintendent of Schools

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6th Grade – CCSS for Mathematics

Grade-Level Introduction

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division, and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

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(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in grade 7 by drawing polygons in the coordinate plane.

FLUENCY

In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., “fluently” multiply multi-digit whole numbers using the standard algorithm (5.NBT.5▲). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary.

The word “fluent” is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.

California *Mathematics Framework,* adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

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Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".

Students:	Teachers:
• Analyze and explain the meaning of the problem • Actively engage in problem solving (Develop, carry out, and refine a plan) • Show patience and positive attitudes • Ask if their answers make sense • Check their answers with a different method	• Pose rich problems and/or ask open ended questions • Provide wait-time for processing/finding solutions • Circulate to pose probing questions and monitor student progress • Provide opportunities and time for cooperative problem solving and reciprocal teaching

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2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

Students:	Teachers:
• Represent a problem with symbols • Explain their thinking • Use numbers flexibly by applying properties of operations and place value • Examine the reasonableness of their answers/calculations	• Ask students to explain their thinking regardless of accuracy • Highlight flexible use of numbers • Facilitate discussion through guided questions and representations • Accept varied solutions/representations

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3. **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

Students:	Teachers:
• Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying and probing questions	• Provide opportunities for students to listen to or read the conclusions and arguments of others • Establish and facilitate a safe environment for discussion • Ask clarifying and probing questions • Avoid giving too much assistance (e.g., providing answers or procedures)

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4. **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

Students:	Teachers:
• Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying questions	• Allow time for the process to take place (model, make graphs, etc.) • Model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) • Make appropriate tools available • Create an emotionally safe environment where risk taking is valued • Provide meaningful, real world, authentic, performance-based tasks (non traditional work problems)

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5. **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

Students:	Teachers:
• Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding	• Make appropriate tools available for learning (calculators, concrete models, digital resources, pencil/paper, compass, protractor, etc.) • Use tools with their instruction

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6. **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.

Students:	Teachers:
• Calculate accurately and efficiently • Explain their thinking using mathematics vocabulary • Use appropriate symbols and specify units of measure	• Recognize and model efficient strategies for computation • Use (and challenging students to use) mathematics vocabulary precisely and consistently

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7. **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

In 6th grade, students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2\ (3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; c=6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.

Students:	Teachers:
• Look for, develop, and generalize relationships and patterns • Apply reasonable thoughts about patterns and properties to new situations	• Provide time for applying and discussing properties • Ask questions about the application of patterns • Highlight different approaches for solving problems

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7. **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

In 6th grade, students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2\ (3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; c=6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.

Students:	Teachers:
• Look for, develop, and generalize relationships and patterns • Apply reasonable thoughts about patterns and properties to new situations	• Provide time for applying and discussing properties • Ask questions about the application of patterns • Highlight different approaches for solving problems

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8. **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Students:	Teachers:
• Look for methods and shortcuts in patterns and repeated calculations • Evaluate the reasonableness of results and solutions	• Provide tasks and problems with patterns • Ask about possible answers before, and reasonableness after computations

6th Grade – CCSS for Mathematics

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6th Grade – CCSS for Mathematics

Grade 6 Overview

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major3 [m] clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. The ▲ symbol will indicate standards in a Major Cluster in the narrative.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade Supporting [s] clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

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6th Grade – CCSS for Mathematics

CCSS Where to Focus Grade 6 Mathematics

Not all of the content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have a greater emphasis is not to say that anything in the standards can be safely neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 6

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters □ Supporting Clusters ○ Additional Clusters

6.RP.A	■	Understand ratio concepts and use ratio reasoning to solve problems.
6.NS.A	■	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
6.NS.B	○	Compute fluently with multi-digit numbers and find common factors and multiples.
6.NS.C	■	Apply and extend previous understandings of numbers to the system of rational numbers.
6.EE.A	■	Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.B	■	Reason about and solve one-variable equations and inequalities.
6.EE.C	■	Represent and analyze quantitative relationships between dependent and independent variables.
6.G.A	□	Solve real-world and mathematical problems involving area, surface area, and volume.
6.SP.A	○	Develop understanding of statistical variability.
6.SP.B	○	Summarize and describe distributions.

REQUIRED FLUENCIES FOR GRADE 6

6.NS.B.2	Multi-digit division
6.NS.B.3	Multi-digit decimal operations

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

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6th Grade – CCSS for Mathematics

6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.

6.RP. 1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

Essential Skills and Concepts:
- ☐ Understand ratio concepts
- ☐ Use ratio language
- ☐ Identify and describe ratio relationships

Question Stems and Prompts:
- ✓ What is a ratio?
- ✓ What is the relationship between these two quantities?
- ✓ What are the different ways a ratio can be written/described?

Vocabulary	**Spanish Cognates**
Tier 2	
• concept	concepto
• relationship	relación
• for every	
• per	por
• for each/for each 1	
Tier 3	
• ratio	proporción
• quantities	cantidades

Standards Connections
4.OA.2, 4.MD.1, 5.NF.5, 5.OA.3 → *6.RP.1*
6.RP.1 → 6.RP.2, 6.RP.3

6.RP.1 Examples:

Examples of Ratio Language
1. If a recipe calls for a ratio of 3 cups of flour to 4 cups of sugar, then the *ratio* of flour to sugar is 3:4. This can also be expressed with units included as "3 cups flour: 4 cups sugar." The associated *rate* is "$\frac{3}{4}$ cups of flour per cup of sugar." The *unit rate* is the number $\frac{3}{4}$ = .75.
2. If the soccer team paid $75 for 15 hamburgers, then this is a *ratio* of $75: 15 hamburgers or 75:15. The associated *rate* is $5 per hamburger. The *unit rate* is the number $\frac{75}{15}$ = 5.

In general, students should be able to identify and describe any ratio using language such as, "For every _____, there are _____." (Adapted from Arizona 2012 and N. Carolina 2012)

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6.RP.A.1

Standard Explanation

A critical area of instruction in grade six is to connect ratio, rate, and percentage to whole number multiplication and division and use concepts of ratio and rate to solve problems. Students' prior understanding of and skill with multiplication, division, and fractions contribute to their study of ratios, proportional relationships, unit rates, and percent in grade six. In grade seven these concepts will extend to include scale drawings, slope, and real-world percent problems.

A ratio is a pair of non-negative numbers, A:B, which are not both zero.[3] In grade six, students learn that ratios are a comparison of two numbers or quantities and that there are two types of ratios—part-to-whole and part-to-part (6.RP. 1▲).

Students work with models to develop their understanding of ratios. (MP.2, MP.6) Initially students do not express ratios using fraction notation so that ratios can be differentiated from fractions and from rates. Later, students understand that ratios can be expressed in fraction notation, but that ratios are different from fractions in several ways.
(CA *Mathematics Framework*, adopted Nov. 6, 2013)

RP Progression Information:

As students work with tables of quantities in equivalent ratios (also called ratio tables), they should practice using and understanding ratio and rate language. It is important for students to focus on the meaning of the terms "for every," "for each," "for each 1," and "per" because these equivalent ways of stating ratios and rates are at the heart of understanding the structure in these tables, providing a foundation for learning about proportional relationships in Grade 7.

6.RP.2 Illustrative Tasks:

- Mangos for Sale, https://www.illustrativemathematics.org/illustrations/77

A store was selling 8 mangos for $10 at the farmers market.

Keisha said,

"That means we can write the ratio 10 : 8, or $1.25 per mango."

Luis said,

"I thought we had to write the ratio the other way, 8 : 10, or 0.8 mangos per dollar."

Can we write different ratios for this situation? Explain why or why not.

- Riding at a Constant Speed, Assessment Variation https://www.illustrativemathematics.org/illustrations/1175

Lin rode a bike 20 miles in 150 minutes. If she rode at a constant speed,

a. How far did she ride in 15 minutes?
b. How long did it take her to ride 6 miles?
c. How fast did she ride in miles per hour?
d. What was her pace in minutes per mile?

6th Grade – CCSS for Mathematics

6.RP.A.1

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6th Grade – CCSS for Mathematics

6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.

6. RP. 2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

Essential Skills and Concepts:
- ☐ Understand unit rate concepts and use these concepts to solve problems
- ☐ Understand that unit rates are associated with ratios
- ☐ Describe unit rates using ratio and rate language

Question Stems and Prompts:
- ✓ What does a unit rate represent?
- ✓ What is the difference between a unit rate and a ratio?

Vocabulary	Spanish Cognates
Tier 3	
• ratio	proporción
• unit rate	

Standards Connections
5.NF.5, 4.OA.2, 5.NF.7 → *6.RP.2*
6.RP.2 → 6.RP.3

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6.RP.A.2

Standard Explanation

Ratios have associated rates. For example, in the ratio 3 cups of orange juice to 2 cups of fizzy water, the rate is 3/2 cups of orange juice per 1 cup of fizzy water. The term unit rate refers to the numerical part of the rate; in the previous example, the unit rate is the number 3/2 = 1.5. (The word "unit" is used to highlight the 1 in "per 1 unit of the second quantity.") Students understand the concept of a unit rate a/b associated with a ratio a:b (with a, b ≠ 0), and use rate language in the context of a ratio relationship (6.RP. 2▲)

Students understand that rates always have units associated with them because they result from dividing two quantities. Common unit rates are cost per item or distance per time. In grade six, the expectation is that student work with unit rates is limited to fractions in which both the numerator and denominator are whole numbers. Grade six students use models and reasoning to find rates and unit rates.

Students understand ratios and their associated rates by building on their prior knowledge of division concepts. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Why must b not be equal to 0?
For a unit rate, or any rational number, $\frac{a}{b}$, the denominator b must not equal 0 since division by 0 is *undefined* in mathematics. To see that division by zero cannot be defined in a meaningful way, we related division to multiplication. That is, if $a \neq 0$ and if $\frac{a}{0} = x$ for some number x, then it must be true that $a = 0 \cdot x$. But since $0 \cdot x = 0$ for any x, there is no x that makes the equation $a = 0 \cdot x$ true. For a

RP Progression Information:

As students work with tables of quantities in equivalent ratios (also called ratio tables), they should practice using and understanding ratio and rate language. It is important for students to focus on the meaning of the terms "for every," "for each," "for each 1," and "per" because these equivalent ways of stating ratios and rates are at the heart of understanding the structure in these tables, providing a foundation for learning about proportional relationships in Grade 7.

6.RP.2 Illustrative Task:

- Games at Recess, https://www.illustrativemathematics.org/illustrations/76

The students in Mr. Hill's class played games at recess.

6 boys played soccer, 4 girls played soccer, 2 boys jumped rope, 8 girls jumped rope

Afterward, Mr. Hill asked the students to compare the boys and girls playing different games.

Mika said, "Four more girls jumped rope than played soccer."

Chaska said, "For every girl that played soccer, two girls jumped rope."

Mr. Hill said, "Mika compared the girls by looking at the difference and Chaska compared the girls using a ratio."

a. Compare the number of boys who played soccer and jumped rope using the difference. Write your answer as a sentence as Mika did.

6th Grade – CCSS for Mathematics

6.RP.A.2

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6th Grade – CCSS for Mathematics

6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Essential Skills and Concepts:

- ☐ Use a variety of strategies (ratio tables, tape diagrams, double number line diagrams, equations) to solve real-world ratio problems
- ☐ Use equivalent ratios to plot values on coordinate plane
- ☐ Find missing values by using tables and diagrams
- ☐ Solve percent problems
- ☐ Use ratio reasoning to convert units

Question Stems and Prompts:

- ✓ How do you know when ratios are equivalent?
- ✓ How can you find the part using the whole?
- ✓ How can you find the whole using the part and a percent?
- ✓ What strategy did you use to solve the ratio problem? Explain your thinking.

Vocabulary	**Spanish Cognates**
Tier 2	
• part	parte
Tier 3	
• rate	
• percent	por ciento
• unit rate	
• tape diagram	
• double number line	

Standards Connections

6.RP.2 → *6.RP.3*
6.RP.3 – 6.EE.7, 6.EE.9

6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Essential Skills and Concepts:

- ☐ Use a variety of strategies (ratio tables, tape diagrams, double number line diagrams, equations) to solve real-world ratio problems
- ☐ Use equivalent ratios to plot values on coordinate plane
- ☐ Find missing values by using tables and diagrams
- ☐ Solve percent problems
- ☐ Use ratio reasoning to convert units

Question Stems and Prompts:

- ✓ How do you know when ratios are equivalent?
- ✓ How can you find the part using the whole?
- ✓ How can you find the whole using the part and a percent?
- ✓ What strategy did you use to solve the ratio problem? Explain your thinking.

Vocabulary	**Spanish Cognates**
Tier 2	
• part	parte
Tier 3	
• rate	
• percent	por ciento
• unit rate	
• tape diagram	
• double number line	

Standards Connections

6.RP.2 → *6.RP.3*
6.RP.3 – 6.EE.7, 6.EE.9

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6.RP.A.3

Standard Explanation

Students make tables of equivalent ratios relating quantities with whole number measurements, they find missing values in the tables, and plot the pairs of values on the coordinate plane. They use tables to compare ratios. (6.RP.3a▲) Grade six students work with tables of quantities in equivalent ratios (also called ratio tables) and practice using ratio and rate language to deepen their understanding of what a ratio describes. As students generate equivalent ratios and record ratios in tables, students should notice the role of multiplication and division in how entries are related to each other. Students also understand that equivalent ratios have the same unit rate. Tables that are arranged vertically can help students see the multiplicative relationship between equivalent ratios, and help avoid confusing ratios with fractions. (Adapted from The University of Arizona Progressions Documents for the Common Core Math Standards [Progressions] 6-7 Ratios and Proportional Relationships [RP] 2011).

New to many sixth grade teachers are tape diagrams and double number line diagrams. A tape diagram expresses a ratio by representing parts with pieces of tape. It is important to note that the units and size of the pieces of tape may not be evident immediately in a given problem (but in a given problem each piece of tape has the same size). Tape diagrams are often used in problems where the two quantities in the ratio have the same units. A double number line diagram sets up two number lines with zeroes connected. The same tick-marks are used on each line, but the number lines have different units, which is central to how double number lines exhibit a ratio. The following table shows how tape diagrams and double number lines can be used to solve the previous example. (Adapted from Progressions on Ratio and Proportion 6-7.) (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6.RP.3 Illustrative Tasks:

- Mixing Concrete, https://www.illustrativemathematics.org/illustrations/53

A mixture of concrete is made up of sand and cement in a ratio of 5 : 3. How many cubic feet of each are needed to make 160 cubic feet of concrete mix?

- Painting a Barn, https://www.illustrativemathematics.org/illustrations/135

Alexis needs to paint the four exterior walls of a large rectangular barn. The length of the barn is 80 feet, the width is 50 feet, and the height is 30 feet. The paint costs $28 per gallon, and each gallon covers 420 square feet. How much will it cost Alexis to paint the barn? Explain your work.

- Dana's House, https://www.illustrativemathematics.org/illustrations/118

The lot that Dana is buying for her new one story house is 35 yards by 50 yards. Dana's house plans show that her house will cover 1,600 square feet of land. What percent of Dana's lot will not be covered by the house? Explain your reasoning.

6th Grade – CCSS for Mathematics

6.RP.A.3

Standard Explanation

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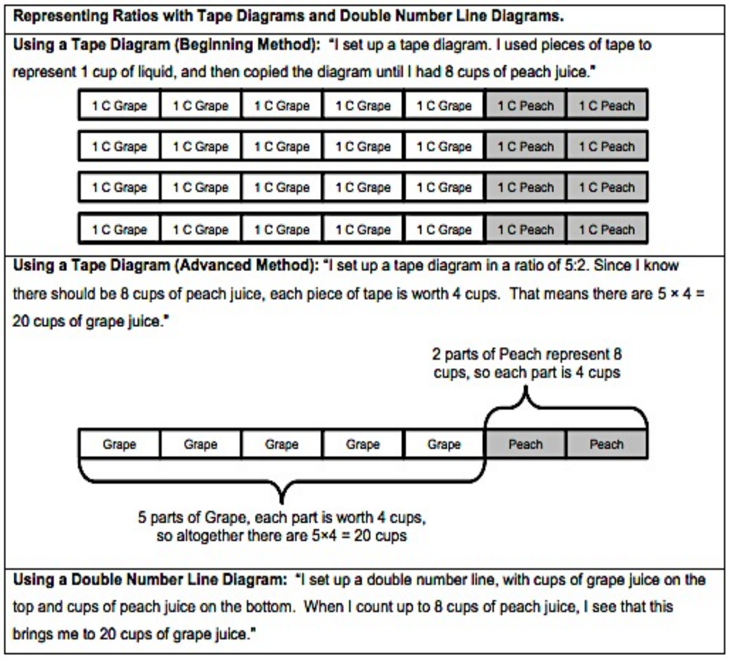
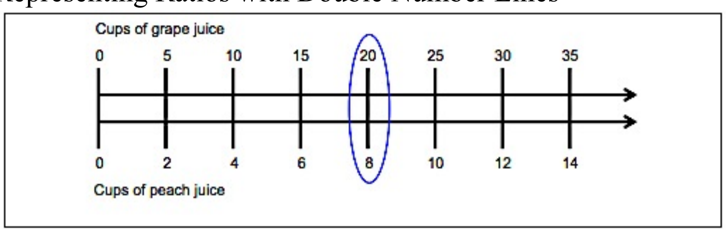
6.RP.A.3 Continued

Standard Explanation

Representing Ratios with Tape Diagrams



Representing Ratios with Double Number Lines



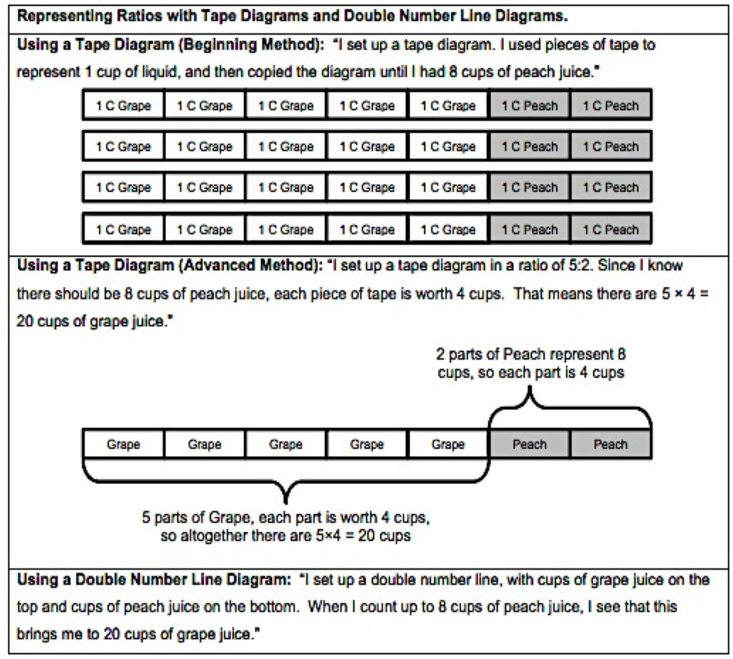
In standard 6.RP.3b-d, students apply their newfound ratio reasoning to various situations in which ratios appear, including unit price, constant speed, percent, and the conversion of measurement units. In sixth grade, generally only whole number ratios are considered. The basic idea of percent is a particularly relevant and important topic for young students to learn, as they will use percent throughout the rest of their lives (MP.4). Percent will be discussed in a separate section that follows, but below are several more examples of ratios and the reasoning expected in the 6.RP domain. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6th Grade – CCSS for Mathematics

6.RP.A.3 Continued

Standard Explanation

Representing Ratios with Tape Diagrams



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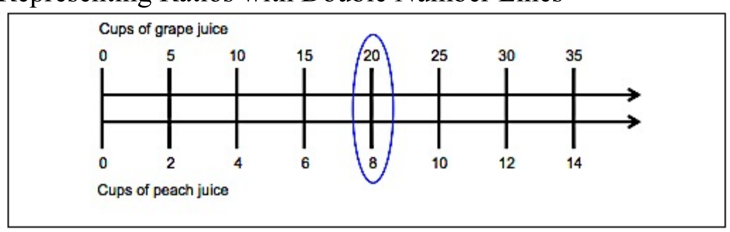

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6.RP.A.3 Continued

Standard Explanation Continued

Representing ratios in various ways can help students see the additive and multiplicative structure of ratios (MP.7). In standard (6.RP.3.a▲), students create tables of equivalent ratios and represent the resulting data on a coordinate grid. Eventually, students see this additive and multiplicative structure in the graphs of ratios, which will be useful later when studying slopes and linear functions. (See also Standard 6.EE.9▲.)

Making Use of Structure in Tables and Graphs of Ratios.

The additive and multiplicative structure of ratios can be pointed out to students in tables as well as graphs. (**6.RP.3.c**)

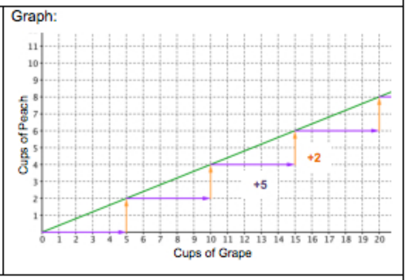
Additive Structure:

Table:

Cups Grape	Cups Peach
5	2
10	4
15	6
20	8
25	10

+5 +5 +5 +5   +2 +2 +2 +2

Graph:



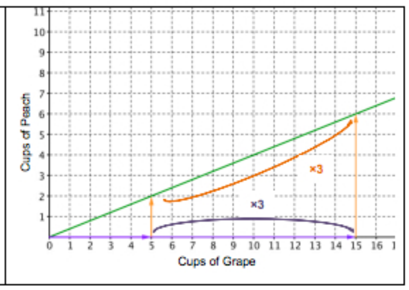
Multiplicative Structure:

Table:

Cups Grape	Cups Peach
5	2
10	4
15	6
20	8
100	40

×2 ×3 ×20   ×2 ×3 ×20

Graph:



(Adapted from Progressions 6-7 RP 2011)

As students solve similar problems they develop several mathematical practice standards as they reason abstractly and quantitatively (MP.2), abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations. Students model with mathematics (MP.4) as they solve the problem with tables and/or ratios. They attend to precision (MP.6) as they properly use ratio notation, symbolism, and label quantities. Following is a sample classroom activity that connects the Standards for Mathematical Content and the Standards for Mathematical Practice, appropriate for students who have already been introduced to ratios and associated rates. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6th Grade – CCSS for Mathematics

6.RP.A.3 Continued

Standard Explanation Continued

Representing ratios in various ways can help students see the additive and multiplicative structure of ratios (MP.7). In standard (6.RP.3.a▲), students create tables of equivalent ratios and represent the resulting data on a coordinate grid. Eventually, students see this additive and multiplicative structure in the graphs of ratios, which will be useful later when studying slopes and linear functions. (See also Standard 6.EE.9▲.)

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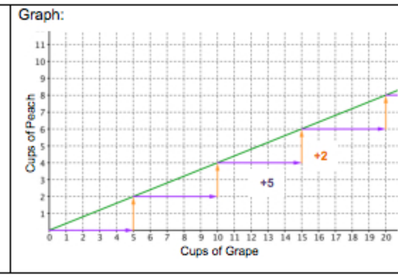

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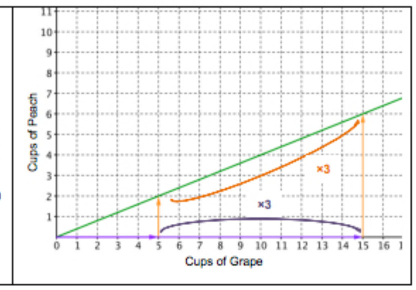

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6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3.c Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Essential Skills and Concepts:

- ☐ Use a variety of strategies (tape diagrams, double number line diagrams, equations) to solve percent problems
- ☐ Explain strategies used, including the parts of tape diagrams and double number lines and what they represent
- ☐ Find the percent of a quantity
- ☐ Find the whole when given a part and the percent

Question Stems and Prompts:

- ✓ How can you find the whole using the part and a percent?
- ✓ What strategy did you use to solve this percent problem?

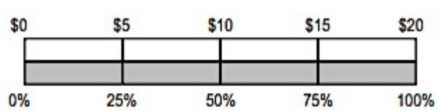
Vocabulary	**Spanish Cognates**
Tier 2	
• part	parte
• whole	
Tier 3	
• rate	
• percent	por ciento
• per 100	
• tape diagram	
• double number line	

6.RP.3.c Example:

Examples of Connecting Percent to Ratio Reasoning.

1. Andrew was given an allowance of $20. He used 75% of his allowance to go to the movies. How much money was spent at the movies?

Solution: "By setting up a percent bar, I can divide the $20 into four equal parts. I see that he spent $15 at the movies."



2. What percent is 12 out of 25?

Part	12	24	48
Whole	25	50	100

Solutions: (a) "I set up a simple table and found that 12 out of 25 is the same as 24 out of 50, which is the same as 48 out of 100. So 12 out of 25 is 48%."

(b) "I saw that 4 × 25 is 100, so I found 4 × 12 = 48. So 12 out of 25 is the same as 48 out of 100, or 48%."

(c) "I know that I can divide 12 by 25, since $\frac{12}{25} = 12 \div 25$. I got .48, which is the same as 48/100, or 48%."

(Adapted from Arizona 2012 and N. Carolina 2012)

6th Grade – CCSS for Mathematics

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- ✓ What strategy did you use to solve this percent problem?

Vocabulary	**Spanish Cognates**
Tier 2	
• part	parte
• whole	
Tier 3	
• rate	
• percent	por ciento
• per 100	
• tape diagram	
• double number line	

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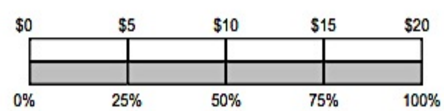

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(Adapted from Arizona 2012 and N. Carolina 2012)

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6.RP.A.3c

Standard Explanation

In Standard 6.RP.3.c, grade six students understand percent as a special type of rate, and they use models and tables to solve percent problems. This is students' first formal introduction to percent. Students understand percentages represent a rate per 100; for example, to find 75% of a quantity means to multiply the quantity by 75/100 or, equivalently, by the fraction ¾. They come to understand this as they represent percent problems with tables, tape diagrams, and double number line diagrams. Student understanding of percent is related to their understanding of fractions and decimals. A thorough understanding of place value helps students see the connection between decimals and percent (for example, students understand that 0.30 represents 30/100, which is the same as 30%).

Students can use simple "benchmark percentages" (e.g., 1%, 10%, 25%, 50%, 75%, or 100%) as one strategy to solve percent problems (e.g., "what is 50% of a number"). By reasoning about rates using the distributive property, students see that percentages can be combined to find other percentages, and thus benchmark percentages become a very useful tool when learning about percent. (MP.5)

A "percent bar" is a visual model, similar to a combined double number line and tape diagram, which can be used to solve percent problems. Students can fold the bar to represent benchmark percentages, such as 50% (half), 25% and 75% (quarters) and 10% (tenths). Teachers should connect percent to ratios, to point out to students that percent is not an unrelated topic, but a useful application of ratios and rates.

There are several types of percent problems that students should become familiar with, including finding the percentage represented by a part out of a whole, finding the unknown part when given a percentage and whole, and finding an unknown whole when a percentage and part are given. Students are not yet responsible for solving multistep percent problems, such as finding sales tax, markups and discounts, or percent change. The following examples illustrate these problem types, as well as how to use tables, tape diagrams, and double number lines to solve them.

When students have had sufficient practice solving percent problems using tables and diagrams, they can be led to representing percentages as decimals as a way to solve problems. In reasoning about and solving percent problems, students develop mathematical practices as they use a variety of strategies to solve problems, use tables and diagrams to represent problems (MP.4), and reason about percent (MP.1, MP.2).

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

6th Grade – CCSS for Mathematics

6.RP.A.3c

Standard Explanation

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(CA *Mathematics Framework*, adopted Nov. 6, 2013)

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6.NS.A Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6. NS. 1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, square mi?*

Essential Skills and Concepts:

- ☐ Solve fraction division problems
- ☐ Interpret the quotient of fractions
- ☐ Use models to represent and explain the problem
- ☐ Create story problems for fraction division problems

Question Stems and Prompts:

- ✓ How can fraction models be used to represent the problem? Explain the parts of your representation.
- ✓ Create a story problem for this division expression. Solve the problem and explain what the quotient means in the context of the problem.
- ✓ What does quotient of the fraction problem mean?

Vocabulary	**Spanish Cognates**
Tier 2	
• interpret	interpretar
Tier 3	
• fraction model	modelo de fracción
• division	división
• equation	ecuación

Standards Connections

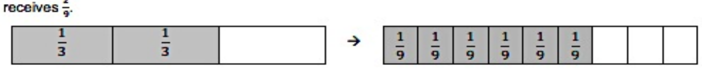
5.NF.7, 3.OA.6 → *6.NS.1*
6.NS.1 → 6.EE.7

6.NS.1 Examples:

Some Examples of Division Reasoning with Fractions.

1. Three people share $\frac{2}{3}$ of a pound of chocolate. How much chocolate does each person get?

Solution: This problem can be represented by $\frac{2}{3} \div 3$. To solve it, students might represent the chocolate with a diagram such as the one below. There are two $\frac{1}{3}$-pound pieces represented in the picture. Students can see that $\frac{1}{3}$ divided among three people is $\frac{1}{9}$. Since there are 2 such pieces, each person receives $\frac{2}{9}$.

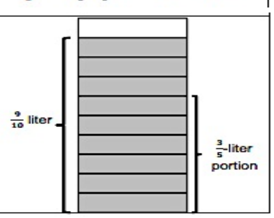


Problems like this one can be used to support the fact that, in general, $\frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c}$.

4. A certain type of water bottle holds $\frac{3}{5}$ of a liter of liquid. How many of these bottles could be filled with $\frac{9}{10}$ of a liter of juice?

Solution: The picture shows $\frac{9}{10}$ of a liter of juice. Since 6 tenths make $\frac{3}{5}$ of a liter, clearly one bottle can be filled. The remaining $\frac{3}{10}$ of a liter represents $\frac{1}{2}$ of a bottle, so it makes sense to say that $1\frac{1}{2}$ bottles could be filled.

Notice that $\frac{1}{10} \div \frac{1}{5} = \frac{1}{2}$, meaning that there is one-half of $\frac{1}{5}$ in each $\frac{1}{10}$. This means that in 9 tenths, there are 9 halves of $\frac{1}{5}$. But



(Adapted from Arizona 2012, N. Carolina 2012, and KATM FlipBook 2012)

6th Grade – CCSS for Mathematics

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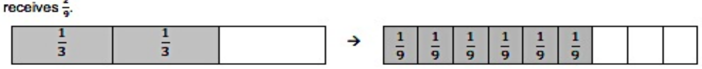
Some Examples of Division Reasoning with Fractions.

1. Three people share $\frac{2}{3}$ of a pound of chocolate. How much chocolate does each person get?

Solution: This problem can be represented by $\frac{2}{3} \div 3$. To solve it, students might represent the chocolate with a diagram such as the one below. There are two $\frac{1}{3}$-pound pieces represented in the picture. Students can see that $\frac{1}{3}$ divided among three people is $\frac{1}{9}$. Since there are 2 such pieces, each person receives $\frac{2}{9}$.

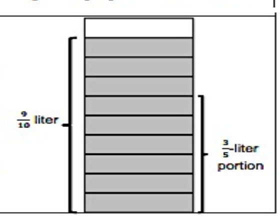


Problems like this one can be used to support the fact that, in general, $\frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c}$.

4. A certain type of water bottle holds $\frac{3}{5}$ of a liter of liquid. How many of these bottles could be filled with $\frac{9}{10}$ of a liter of juice?

Solution: The picture shows $\frac{9}{10}$ of a liter of juice. Since 6 tenths make $\frac{3}{5}$ of a liter, clearly one bottle can be filled. The remaining $\frac{3}{10}$ of a liter represents $\frac{1}{2}$ of a bottle, so it makes sense to say that $1\frac{1}{2}$ bottles could be filled.

Notice that $\frac{1}{10} \div \frac{1}{5} = \frac{1}{2}$, meaning that there is one-half of $\frac{1}{5}$ in each $\frac{1}{10}$. This means that in 9 tenths, there are 9 halves of $\frac{1}{5}$. But



(Adapted from Arizona 2012, N. Carolina 2012, and KATM FlipBook 2012)

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6th Grade – CCSS for Mathematics

6.NS.A.1

Standard Explanation

In grade six students complete their understanding of division of fractions and extend the notion of number to the system of rational numbers, which includes negative numbers. Students also work toward fluency with multi-digit division and multi-digit decimal operations.

In grade five students learned to divide whole numbers by unit fractions and unit fractions by whole numbers. These experiences lay the conceptual foundation for understanding general division of fractions in sixth grade. Grade six students continue to develop division by using visual models and equations to divide fractions by fractions to solve word problems (6.NS.1▲). Student understanding of the meaning of the operations with fractions builds upon the familiar understandings of these meanings with whole numbers and can be supported with visual representations. To help students make this connection, teachers might have students think about a simpler problem with whole numbers and then use the same operation to solve with fractions.

Looking at the problem through the lens of “How many groups?” or “How many in each group?” helps students visualize what is being sought. Encourage students to explain their thinking and to recognize division in two different situations—measurement division, which requires finding how many groups (e.g., how many groups can you make?) and fair-share division, which requires equal sharing (e.g., finding how many are in each group). In fifth grade, students represented division problems like with diagrams and reasoned why the answer is (e.g. how many halves are in 4?). They may have discovered that can be found by multiplying (i.e., each whole gives 2 halves, so there are 8 halves altogether). Similarly, students may have found that. These generalizations will be exploited when developing general methods for dividing fractions. Teachers should be aware that making visual models for general division of fractions can be difficult; it can be simpler to move to discussing general methods of dividing fractions and use one of these methods to solve problems. Students should develop reasoning about fraction division, before moving to general methods.

Common Misconception: Students may confuse dividing a quantity **by** $\frac{1}{2}$ with dividing a quantity **in half**. Dividing **by** $\frac{1}{2}$ is finding how many $\frac{1}{2}$-sized portions there are, as in "dividing 7 **by** $\frac{1}{2}$," which is $7 \div \frac{1}{2} = 14$. On the other hand, to divide a quantity **in half** is to divide the quantity into two parts equally, as in "dividing 7 **in half**" yields $\frac{7}{2} = 3.5$. Students should understand that dividing **in half** is the same as dividing by 2. (Adapted from KATM 6th FlipBook 2012)

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

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6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS. 2 Fluently divide multi-digit numbers using the standard algorithm.

Essential Skills and Concepts:
- ☐ Divide multi-digit numbers using strategies and the standard algorithm
- ☐ Explain how the standard algorithm works relating it to other strategies
- ☐ Fluently divide multi-digit numbers

Question Stems and Prompts:
- ✓ How can knowing factors and multiples help you to fluently divide?
- ✓ How can you use your place value understanding when you divide?
- ✓ How can you use the relationship between multiplication and division to help you divide?
- ✓ How can you relate your work with other methods such as partial quotients and the area model to the standard algorithm?

Vocabulary	**Spanish Cognates**
Tier 2	
• fluently	fluidez
• estimate	estimación
• relationship	relación
Tier 3	
• multiple	múltiplo
• factor	factor
• partial quotients	cociente parcial
• area model	modelo de área
• standard algorithm	algoritmo estándar

Standards Connections
5.NBT.6 → *6.NS.2*
6.NS.2 → 6.NS.3

6.NS.2 Example:

Example. Division using single digits instead of totals.

If writing only single digits, being attentive to place value language, teachers can ask, "how many groups of 16 are in 34 (hundreds)?" Since there are two groups of 16 in 34, there are 2 (hundred) groups of 16 in 34 (hundreds), so we record this with a 2 in the hundreds place above the dividend. The product of 2 and 16 is recorded, and we subtract 32 from 34, understanding that we are subtracting 32 hundreds from 34 hundreds, yielding 2 hundreds remaining. Next, when we "bring the 4 down to write 24," we understand this as moving to the digit in the dividend necessary to obtain a number larger than the divisor. Again, we focus on the fact that there are 24 (tens) remaining, and so the question becomes, "How many groups of 16 are in 24 tens?" The algorithm continues and the quotient is found.

6th Grade – CCSS for Mathematics

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6th Grade – CCSS for Mathematics

6.NS.B.2

Standard Explanation

In previous grades, students built a conceptual understanding of operations with whole numbers and became fluent in multi-digit addition, subtraction, and multiplication. In grade six, students work toward fluency with multi-digit division and multi-digit decimal operations (6.NS.2-3). Fluency with the standard algorithms is expected, but an algorithm is defined by its steps and not the way those steps are recorded in writing, so minor variations in written methods are acceptable.

Grade six students fluently divide using the standards algorithm. Students should examine several methods to record division of multi-digit numbers and focus on a variation of the standard algorithm that is efficient and that makes sense to them. They can compare variations to understand how the same step can be written differently but still have the same place value meaning. All such discussions should include place value terms. Students should see examples of standard algorithm division that can be easily connected to place value meanings.

The partial quotients can also be written above each other over the dividend. Students can also consider writing single digits instead of totals, provided they can explain why they do so with place value reasoning, dropping all of the 0s in the quotients and subtractions in the dividend; they then write 215 step-by-step above the dividend. In both cases, students use place value reasoning.

Students should have experience with many examples such as the previous one and the one that follows. Teachers should be prepared to support discussions involving place value should misunderstanding arise. There may be other effective ways to include place value concepts in explaining a variation of the standard algorithm for division, and teachers are encouraged to find a method that works for them and their students. Overall, teachers should remember that the standards support coherence of learning and conceptual understanding, and instruction that builds on students' previous mathematical experiences is crucial.

To be prepared for grade seven mathematics students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade six and have met the fluency expectations for the grade six. For sixth graders, the expected fluencies are multi-digit whole number division (6.NS.2) and multi-digit decimal operations (6.NS.3). These fluencies and the conceptual understandings that support them are foundational for work with fractions and decimals in grade seven. (*CA Mathematics Framework*, adopted Nov. 6, 2013)

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6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Essential Skills and Concepts:
- ☐ Add, subtract, multiply, and divide multi-digit decimals using strategies and the standard algorithm
- ☐ Explain how the standard algorithm works relating it to other strategies or visual models
- ☐ Fluently add, subtract, multiply, and divide multi-digit decimals

Question Stems and Prompts:
- ✓ How is adding/subtracting whole numbers similar and different to adding/subtracting with decimals?
- ✓ How is multiplying/dividing with whole numbers similar and different to multiplying/dividing with decimals?
- ✓ Explain your strategy for solving. How might you complete this problem in another way?

Vocabulary	**Spanish Cognates**
Tier 3	
• decimal	decimal
• decimal point	punto decimal

Standards Connections
5.NBT.5, 5.NBT.7, 5.NBT.6, 6.NS.2 → *6.NS.3*

6.NS.3 Examples:

Examples of Decimal Operations.

1. Maria had 3 kilograms of sand for a science experiment. She had to measure out exactly 1.625 kilograms for a sample. How much sand will be left after she measures out the sample?

Solution: Student thinks, "I know that 1.625 is a little more than 1.5, so I should have about 1.5 kilograms remaining. I need to subtract common place values from each other, and I notice that 1.625 has three place values to the right of the ones place, so if I make 0s in the tenths, hundredths and thousandths places of 3 to make 3.000, then the numbers have the same number of place values. Then it's easier to subtract: 3.000 – 1.625 = 1.375. There are 1.375 kilograms left."

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

The Task

Sarah and three friends have decided to go to a Ravens game at M&T Bank Stadium. They are discussing the best route to take to carpool and get to the stadium. You have researched three possible routes in order to choose the best route to take.

Route #1:
Sarah and Justin will meet at the Snowden River Park and Ride. From there they will travel to the Arbutus Park and Ride to meet Dante and Chelsea. Next the group will travel from the Arbutus Park and Ride to the Stadium. (To view Google Map: http://goo.gl/maps/DhUWi)

Route #2:
Sarah and Justin will meet at the Snowden River Park and Ride. From there they will travel to Lansdowne High School to meet Dante and Chelsea. Next the group will travel from Lansdowne High School to the Stadium. (To view Google Map: http://goo.gl/maps/24GOI)

Route #3:
Sarah and Justin will meet at the Snowden River Park and Ride. From there they will travel to Dante's house across the street from Arbutus Elementary School. Next the group will travel from Dante's house to Chelsea's house in Baltimore Highlands. The group will leave Chelsea's house and travel to the Stadium. (To view Google Map: http://goo.gl/maps/5pFfS)

Use the three maps provided in order to decide which route you should take and why? Determine the total distance and time traveled. Take into account any and all factors that may influence your decision.

6th Grade – CCSS for Mathematics

6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

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Vocabulary / Spanish Cognates
Tier 3
• decimal — decimal
• decimal point — punto decimal

Standards Connections
5.NBT.5, 5.NBT.7, 5.NBT.6, 6.NS.2 → 6.NS.3

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6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Standard Explanation

In grade five students multiplied decimals to hundredths. They understood that multiplying by decimals moves the decimal point as many places to the right as there are places in the multiplying decimal. In grade six students extend and apply their place value understanding to fluently multiply multi-digit decimals (6.NS.3). Writing decimals as fractions whose denominator is a power of 10 can be used to explain the "decimal point rule" in multiplication. This logical reasoning based on place value and decimal fractions justifies the typical rule, "count the decimal places in the numbers and insert the decimal point to make that many places in the product."

The general methods used for computing quotients of whole numbers extend to decimals with the additional concern of where to place the decimal point in the quotient. Students have experienced dividing decimals to hundredths in grade five, but in grade six they move to using standard algorithms for doing so. In simpler cases, like with 16.8 415 ÷ 8, students can simply apply the typical division algorithm, paying mind to place value. When problems get more difficult, e.g., when the divisor also has a decimal point, then students may need to use strategies involving rewriting the problem through changing place values. Reasoning similar to that for multiplication can be used to explain the rule that "when the decimal point in the divisor is moved to make a whole number, the decimal point in the dividend should be moved the same number of places." For example, for a problem like 4.2 ÷ .35, wherein someone might give a rote recipe: "move the decimal point two places to the right" in .35 and also in 4.2, teachers can instead appeal to the idea that one can make a simpler but equivalent division problem by multiplying both numbers by 100 and still obtain the same quotient. That is,

$$4.2 \div .35 = (4.2 \times 100) \div (.35 \times 100) = 420 \div 35 = 12.$$

Attention to student understanding of place value is of the utmost importance. There is no conceptual understanding gained by referring to this only as "moving the decimal point," teachers can refer to this more meaningfully as "multiplying by n/n in the form of 100/100." (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6th Grade – CCSS for Mathematics

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6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Essential Skills and Concepts:
- ☐ Find the greatest common factor of two whole numbers less than 100
- ☐ Find the least common multiple of two whole numbers less than or equal to 12
- ☐ Express the sum of two numbers using the distributive property

Question Stems and Prompts:
- ✓ What is the difference between the GCF and LCM?
- ✓ Why does using the distributive property create an equivalent expression? Justify your thinking.

Vocabulary	**Spanish Cognates**
Tier 3	
• least common multiple (LCM)	
• greatest common factor (GCF)	
• distributive property	propiedad distributiva

Standards Connections
4.OA.4, 5.OA.2 → *6.NS.4, 6.NS.4* → 6.EE.3, 6.EE.4

6.NS.4 Examples:

Example: Ladder Method for Finding GCF and LCM

To find the LCM and GCF of 120 and 48, one can use the "ladder method," which systematically finds common factors of 120 and 48, and leaves us with the factors that 120 and 48 do not have in common. The GCF becomes the product of all those factors that 120 and 48 share, while the LCM is the product of the GCF and the remaining uncommon factors of 120 and 48. With the ladder method, common factors (3, 4, 2 in this case) are divided from the starting and remaining numbers until no more common factors to divide (5, 2). The GCF is then $3 \cdot 4 \cdot 2 = 24$ while the LCM is $24 \cdot 5 \cdot 2 = 240$.

Common Factors	Remaining Numbers	
3	120	48
4	40	16
2	10	4
	5	2

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

The Task

A popular game show is creating a spin-off of its show where each game is won by using various math skills. The producers of the show want your class to test out a potential game. In this game there are four prizes you can win, each prize is labeled with its price and an addition problem. In order to win the game, you have to first solve each addition problem and then, using only the numbers in the price of the prize, create another math problem whose answer is the same as that of the addition problem. Each correct match wins you that prize! Correctly matching all four wins you all the prizes and a bonus cash prize!
Here are the four prizes you could win. Good Luck!

Cruise to the Bahamas	**A New iPad**
$942	**$583**
36 + 8	**15 + 24**
____________	____________

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6th Grade – CCSS for Mathematics

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Standards Connections
4.OA.4, 5.OA.2 → *6.NS.4, 6.NS.4* → 6.EE.3, 6.EE.4

6.NS.4 Examples:

Example: Ladder Method for Finding GCF and LCM

To find the LCM and GCF of 120 and 48, one can use the "ladder method," which systematically finds common factors of 120 and 48, and leaves us with the factors that 120 and 48 do not have in common. The GCF becomes the product of all those factors that 120 and 48 share, while the LCM is the product of the GCF and the remaining uncommon factors of 120 and 48. With the ladder method, common factors (3, 4, 2 in this case) are divided from the starting and remaining numbers until no more common factors to divide (5, 2). The GCF is then $3 \cdot 4 \cdot 2 = 24$ while the LCM is $24 \cdot 5 \cdot 2 = 240$.

Common Factors	Remaining Numbers	
3	120	48
4	40	16
2	10	4
	5	2

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

The Task

A popular game show is creating a spin-off of its show where each game is won by using various math skills. The producers of the show want your class to test out a potential game. In this game there are four prizes you can win, each prize is labeled with its price and an addition problem. In order to win the game, you have to first solve each addition problem and then, using only the numbers in the price of the prize, create another math problem whose answer is the same as that of the addition problem. Each correct match wins you that prize! Correctly matching all four wins you all the prizes and a bonus cash prize!
Here are the four prizes you could win. Good Luck!

Cruise to the Bahamas	**A New iPad**
$942	**$583**
36 + 8	**15 + 24**
____________	____________

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6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Standard Explanation

In fourth grade students identified prime numbers, composite numbers, and factor pairs. In sixth grade students build on prior knowledge and find the greatest common factor (GCF) of two whole numbers less than or equal to 100 and find the least common multiple (LCM) of two whole numbers less than or equal to 12 (6.NS.4). Teachers might employ compact methods for finding the LCM and GCF of two numbers, such as the ladder method illustrated below, among other methods (such as listing multiples of each number and identifying the least they have in common for LCM, etc.).
(CA *Mathematics Framework*, adopted Nov. 6, 2013)

6.NS.4 Illustrative Task:

- The Florist Shop, https://www.illustrativemathematics.org/illustrations/259

The florist can order roses in bunches of one dozen and lilies in bunches of 8. Last month she ordered the same number of roses as lilies. If she ordered no more than 100 roses, how many bunches of each could she have ordered? What is the smallest number of bunches of each that she could have ordered? Explain your reasoning.

- Adding Multiples, https://www.illustrativemathematics.org/illustrations/257

Nina was finding multiples of 6. She said,

18 and 42 are both multiples of 6, and when I add them, I also get a multiple of 6:

$$18 + 42 = 60.$$

Explain to Nina why adding two multiples of 6 will always result in another multiple of 6.

- Factors and Common Factors, https://www.illustrativemathematics.org/illustrations/255

a. List all the factors of 48.
b. List all the factors of 64.
c. What are the common factors of 48 and 64?
d. What is the greatest common factor of 48 and 64?

- Multiples and Common Multiples, https://www.illustrativemathematics.org/illustrations/256

a. List all the multiples of 8 that are less than or equal to 100.
b. List all the multiples of 12 that are less than or equal to 100.
c. What are the common multiples of 8 and 12 from the two lists?
d. What is the least common multiple of 8 and 12?
e. Lyle noticed that the list of common multiples has a pattern. Describe a pattern in the list of numbers that Lyle might have seen.

6th Grade – CCSS for Mathematics

6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Standard Explanation

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6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Essential Skills and Concepts:
- ☐ Understand that positive and negative numbers are used to describe quantities with opposite values and directions
- ☐ Represent real word situations using positive and negative numbers and zero

Question Stems and Prompts:
- ✓ Create a list of real world examples that illustrate positive and negative numbers.
- ✓ What real-world situations can be represented by 0?
- ✓ How are positive and negative numbers related?

Vocabulary	**Spanish Cognates**
Tier 2	
• positive	positivo
• negative	negativo
• opposite	opuesto
• value	valor

Standards Connections
6.NS.5 → 6.NS.6

6.NS.5 Examples:

Examples of Rational Numbers in Context.

1. All substances are made up of atoms, and atoms have protons and electrons. A proton has a "positive charge", represented by "+1," and an electron has a "negative charge," represented by "−1." A group of 5 protons has a total charge of +5 and a group of 8 electrons has a total charge of −8. One positive charge combines with one negative charge to result in a "neutral charge", which we can represent by $(+1) + (-1) = 0$. So for example, a group of 4 protons and 4 electrons together would have a neutral charge since there are 4 positive charges to combine with 4 negative charges. We could write this as: $(+4) + (-4) = 0$.
   a. What is the overall charge of a group of 3 protons and 3 electrons?
   b. What is the overall charge of a group of 5 protons with no electrons?
   c. What is the overall charge of a group of 4 electrons with no protons?
2. In a checking account, "credits" to the account are recorded as positive numbers (since they are adding money to the account), and "debits" to the account are recorded as negative numbers (since they are taking away money from the account).
   a. Explain the meaning of an account statement that reads a total balance of −$100.15.
   b. Explain the meaning of an account statement that reads a total balance of $225.78.
   c. If someone's bank statement reads −$45.67, then explain how they can get to a $0 balance.

(CA *Mathematics Framework*, adopted Nov. 6, 2013

6th Grade – CCSS for Mathematics

6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

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6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6. NS. 5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Standard Explanation

In grade six students begin the formal study of negative numbers, their relationship to positive numbers, and the meaning and uses of absolute value. Students use rational numbers (expressed as fractions, decimals, and integers) to represent real-world contexts and they understand the meaning of 0 in each situation (6.NS.5▲). Count models (positive/negative electric charge, credits/debits), where the context gives rise to the basic meaning of, will help develop student understanding of the relationship between a number and its opposite. In addition, measurement contexts such as temperature and elevation can contribute to student understanding of these ideas. (MP.1, MP.2, MP.4)

Note that the standards do not specifically mention the set of the integers (which consists of the whole numbers and their opposites) as a distinct set of numbers. Rather, the standards are focused on student understanding of the set of rational numbers in general (which consists of whole numbers, fractions, and their opposites). Thus, while early instruction in positives and negatives will likely start with examining whole numbers and their opposites, students must also experience working with negative fractions (and decimals) at this grade level. Ultimately, students learn that all numbers have an "opposite." (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6.NS.5 Illustrative Tasks:

- It's Warmer in Miami, https://www.illustrativemathematics.org/illustrations/277

One morning the temperature is -28° in Anchorage, Alaska, and 65° in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?

- Mile High, https://www.illustrativemathematics.org/illustrations/278

Denver, Colorado is called "The Mile High City" because its elevation is 5280 feet above sea level. Someone tells you that the elevation of Death Valley, California is −282 feet.

a. Is Death Valley located above or below sea level? Explain.

b. How many feet higher is Denver than Death Valley?

c. What would your elevation be if you were standing near the ocean?

6th Grade – CCSS for Mathematics

6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6. NS. 5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

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6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Essential Skills and Concepts:
- ☐ Understand that rational numbers are points on a number line
- ☐ Extend number lines and the coordinate plane to include negative numbers and negative coordinates
- ☐ Recognize and plot opposite numbers on a number line
- ☐ Understand that the signs of the numbers in ordered pairs represent the location of the ordered pair
- ☐ Plot coordinate pairs in all four quadrants of the coordinate plane

Question Stems and Prompts:
- ✓ How do you find the opposite of a given integer?
- ✓ Describe the location of opposites.
- ✓ How do the signs of the ordered pairs determine the quadrant they are located in on the coordinate plane?
- ✓ Given two coordinates with the same numerals, but different signs, how would you know their locations in the coordinate plane?

Vocabulary	**Spanish Cognates**
Tier 2	
• horizontal	horizontal
• vertical	vertical
• opposite	opuesto
Tier 3	
• quadrant	cuadrante

Standards Connections
6.NS.6a → 6.EE.8, 6.NS.6b – c, 6.NS.7c
6.NS.6b → 6.NS.8, 6.NS.6b – 6.NS.6c
6.NS.6c → 6.NS.7b, 6.NS.7b

6th Grade – CCSS for Mathematics

6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

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a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
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Essential Skills and Concepts:
- ☐ Understand that rational numbers are points on a number line
- ☐ Extend number lines and the coordinate plane to include negative numbers and negative coordinates
- ☐ Recognize and plot opposite numbers on a number line
- ☐ Understand that the signs of the numbers in ordered pairs represent the location of the ordered pair
- ☐ Plot coordinate pairs in all four quadrants of the coordinate plane

Question Stems and Prompts:
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Vocabulary / **Spanish Cognates**
Tier 2
• horizontal — horizontal
• vertical — vertical
• opposite — opuesto
Tier 3
• quadrant — cuadrante

Standards Connections
6.NS.6a → 6.EE.8, 6.NS.6b – c, 6.NS.7c
6.NS.6b → 6.NS.8, 6.NS.6b – 6.NS.6c
6.NS.6c → 6.NS.7b, 6.NS.7b

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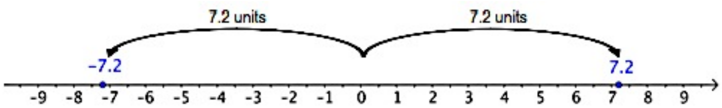
6th Grade – CCSS for Mathematics

6.NS.C.6

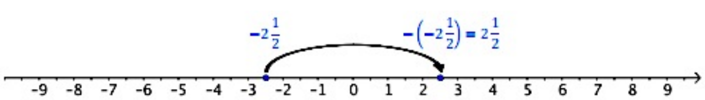
Standard Explanation

In prior grades students worked with positive fractions, decimals, and whole numbers on the number line and in the first quadrant of the coordinate plane. In sixth grade, students extend the number line to represent all rational numbers focusing on the relationship between a number and its opposite, namely that they are equidistant from 0 on a number line. (6.NS.6▲) Number lines may be either horizontal or vertical (such as on a thermometer); experiencing both will facilitate students' movement from number lines to coordinate grids.

In grade seven, students will explore operations with positive and negative rational numbers, so it is important to develop a firm understanding of the relationship between positive and negative numbers and their opposites here. Students recognize that a number and its opposite are the same distance from 0 on a number line, as in 7.2 and – 7.2 being the same distance from 0:



In addition, students recognize the minus sign as meaning "the opposite of," and that in general the opposite of a number is the number on the other side of 0 at the same distance from 0 as the original number, as in – (– 2 ½) is "the opposite of the opposite of – 2 ½," which is just 2 ½ again:



This understanding will help with later development of the notion of absolute value, as the absolute value of a number is defined as its distance from 0 on a number line. Students' previous work in the first quadrant grid helps them recognize the point where the -axis and -axis intersect as the origin. Grade six students identify the four quadrants and the appropriate quadrant for an ordered pair based on the signs of the coordinates (6.NS.6▲). For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (–, +). Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (−2, 4) and (−2, −4), the -coordinates differ only by signs, which represents a reflection across the -axis. A change in the -coordinates from (−2, 4) to (2, 4), represents a reflection across the -axis. When the signs of both coordinates change, for example, when (2, −4) changes to (−2, 4), the ordered pair is reflected across both axes. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6.NS.C.6

Standard Explanation

In prior grades students worked with positive fractions, decimals, and whole numbers on the number line and in the first quadrant of the coordinate plane. In sixth grade, students extend the number line to represent all rational numbers focusing on the relationship between a number and its opposite, namely that they are equidistant from 0 on a number line. (6.NS.6▲) Number lines may be either horizontal or vertical (such as on a thermometer); experiencing both will facilitate students' movement from number lines to coordinate grids.

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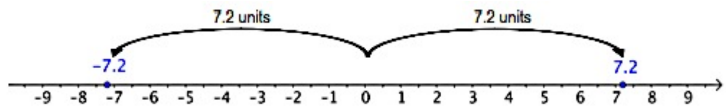

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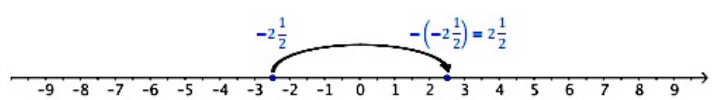

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6th Grade – CCSS for Mathematics

6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6. NS. 7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*
c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

Essential Skills and Concepts:

- ☐ Order rational numbers
- ☐ Understand absolute value of rational numbers
- ☐ Understand absolute as the distance a number is from 0
- ☐ Interpret and explain inequality statements and apply them to real world situations
- ☐ Understand and explain the difference between absolute value and order statements

Question Stems and Prompts:

- ✓ Which rational number is greater/less? Justify your thoughts using a number line diagram.
- ✓ Explain the meaning of absolute value in your own words.
- ✓ Write and explain an inequality that represents a real world situation.

Vocabulary	**Spanish Cognates**
Tier 2	
• order	ordenar
Tier 3	
• absolute value	valor absoluto
• number line	línea de números
• inequality	

Standards Connections

6.NS.7c → 6.NS.7d, 6.NS.7b – 6.NS.7a
6.NS.7a, 6.NS.7b →6.NS.7d

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Vocabulary	**Spanish Cognates**
Tier 2	
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Tier 3	
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Standards Connections

6.NS.7c → 6.NS.7d, 6.NS.7b – 6.NS.7a
6.NS.7a, 6.NS.7b →6.NS.7d

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6.NS.C.7

Standard Explanation

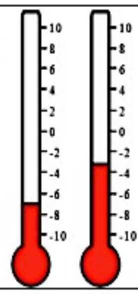
In grade six students reason about the order and absolute value of rational numbers (6.NS.7▲) and solve real world and mathematical problems by graphing in all four quadrants of the coordinate plane (6.NS.8▲). Students use inequalities to express the relationship between two rational numbers. Working with number line models helps students internalize the order of the numbers—larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. Students correctly locate rational numbers on the number line, write inequalities, and explain the relationships between numbers. Students understand the absolute value of a rational number as its distance from 0 on the number line and recognize the symbols "| |" as representing absolute value (e.g., | 3 | = 3, | -2 | = 2). They distinguish comparisons of absolute value from statements about order. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6.NS.7 Examples:

Example: Comparing Rational Numbers.

One of the thermometers shows −3°C and the other shows −7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.

Solution: Since on a vertical number line, negative numbers get "more negative" as we go down the line, it appears that the thermometer on the left must read −7 and the thermometer on the right must read −3. By counting spaces they differ by, the thermometer on the left reads a temperature colder by 4 degrees. Related inequalities are −7 < −3 and −3 > −7.



(CA *Mathematics Framework*, adopted Nov. 6, 2013)

Common Misconceptions: With positive numbers the absolute value (distance from zero) of the number and the value of the number are the same. However students might be confused when they work with the absolute value of negative numbers. For negative numbers, as the value of the number decreases the absolute value increases. For example −24 is less than −14 because −24 is located to the left of −14 on the number line. However the absolute value of −24 is greater than the absolute value of −14. Students may also erroneously think that taking the absolute value means to "change the sign of a number" which is true for negative numbers but not for positive numbers or 0.

(Adapted from Arizona 2012, N. Carolina 2012, and KATM FlipBook 2012)

6.NS.7 Illustrative Task:

- Above and Below Sea Level, https://www.illustrativemathematics.org/illustrations/288

The table below shows the lowest elevation above sea level in three American cities.

City	State	Elevation above sea level	Elevation below sea level
Denver	Colorado	5130	
New Orleans	Louisiana	-8	
Seattle	Washington	0	

Finish filling in the table as you think about the following statements. Decide whether each of the following statements is true or false. Explain your answer for each one.

a. True or False? New Orleans is $|-8|$ feet below sea level.

b. True or False? New Orleans is −8 feet below sea level.

c. True or False? New Orleans is 8 feet below sea level.

d. True or False? Seattle is 0 feet above sea level.

6th Grade – CCSS for Mathematics

6.NS.C.7

Standard Explanation

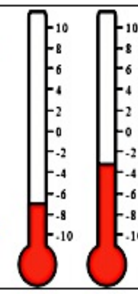
In grade six students reason about the order and absolute value of rational numbers (6.NS.7▲) and solve real world and mathematical problems by graphing in all four quadrants of the coordinate plane (6.NS.8▲). Students use inequalities to express the relationship between two rational numbers. Working with number line models helps students internalize the order of the numbers—larger numbers on the right or top of the number line and smaller numbers to the left or bottom of the number line. Students correctly locate rational numbers on the number line, write inequalities, and explain the relationships between numbers. Students understand the absolute value of a rational number as its distance from 0 on the number line and recognize the symbols "| |" as representing absolute value (e.g., | 3 | = 3, | -2 | = 2). They distinguish comparisons of absolute value from statements about order. (CA Mathematics Framework, adopted Nov. 6, 2013)

6.NS.7 Examples:

Example: Comparing Rational Numbers.

One of the thermometers shows −3°C and the other shows −7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.

Solution: Since on a vertical number line, negative numbers get "more negative" as we go down the line, it appears that the thermometer on the left must read −7 and the thermometer on the right must read −3. By counting spaces they differ by, the thermometer on the left reads a temperature colder by 4 degrees. Related inequalities are −7 < −3 and −3 > −7.



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6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Essential Skills and Concepts:
- ☐ Graph points in all four quadrants of the coordinate plane
- ☐ Find the distances of the points by using coordinates and the absolute value

Question Stems and Prompts:
- ✓ What is the distance between the two points?
- ✓ Why do we use the absolute value to find the distance between two points?

Math Vocabulary	**Spanish Cognates**
Tier 2	
• points	punto
• distance	distancia
Tier 3	
• quadrant	cuadrante
• coordinate pairs	coordenadas

Standards Connections
6.NS.8 – 6.G.3

6.NS.8 Example:

<u>Example 1:</u>
What is the distance between (–5, 2) and (–9, 2)?
Solution: **The distance would be 4 units. This would be a horizontal line since the *y*-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances 9 and 5. (| 9 | - | 5 |). Coordinates could also be in two quadrants and include**

(North Carolina Unpacking Document, July 2013)

6th Grade – CCSS for Mathematics

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Math Vocabulary	**Spanish Cognates**
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Standards Connections
6.NS.8 – 6.G.3

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6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS. 8
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

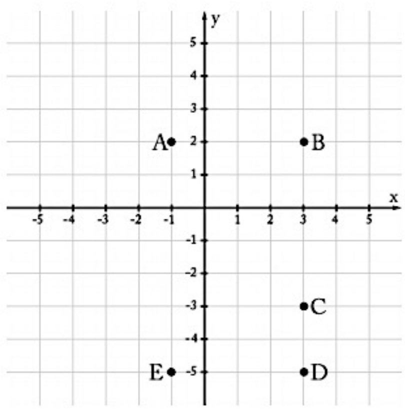
Standard Explanation

Students plot points within all four quadrants of the coordinate plane to solve both real world and mathematical problems. They find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). This builds on their work with absolute values in 6.NS.7. (Adapted from the North Carolina Unpacking Document, July 2013)

6.NS.8 Illustrative Task:

- Distance between Points, **https://www.illustrativemathematics.org/illustrations/290**

Some points are shown in the coordinate plane below.



a. What is the distance between points B & C?

b. What is the distance between points D & B?

c. What is the distance between points D & E?

d. Which of the points shown above are 4 units away from $(-1, -3)$ and 2 units away from $(3, -1)$?

6th Grade – CCSS for Mathematics

6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS. 8
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Standard Explanation

Students plot points within all four quadrants of the coordinate plane to solve both real world and mathematical problems. They find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). This builds on their work with absolute values in 6.NS.7. (Adapted from the North Carolina Unpacking Document, July 2013)

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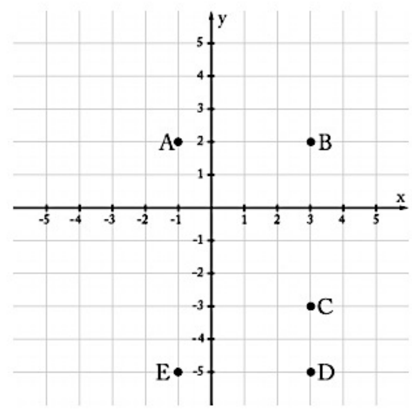

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6th Grade – CCSS for Mathematics

6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Essential Skills and Concepts:
- ☐ Write numerical expression involving whole number exponents
- ☐ Evaluate numerical expressions involving whole number exponents

Question Stems and Prompts:
- ✓ What do exponents represent?
- ✓ Expand the power to show its meaning before evaluating it. How does this help you to understand the meaning of exponents?
- ✓ Describe how to evaluate an expression with exponents.

Vocabulary	**Spanish Cognates**
Tier 2	
• base	base
• powers	
Tier 3	
• numerical expression	expresión numérica
• exponents	exponente

Standards Connections
6.EE.1 → 6.EE.2

6.EE.1 Examples:

Examples:
• What is the side length of a cube of volume 5^3 cubic cm? (5 cm)
• Write $10{,}000 = 10 \times 10 \times 10 \times 10$ with an exponent. (10^4)
• Andrea had half a pizza. She gave half of it to Marcus. Then Marcus gave half of what he had to Roger. Write the amount of pizza Roger has using exponents. $\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\left(\frac{1}{2}\right)^3\right)$
Evaluate the Following:
• 4^3 (Answer: $4 \times 4 \times 4 = 64$)
• $5 + 2^4 \cdot 6$ (Answer: $5 + 16 \cdot 6 = 5 + 96 = 101$)
• $7^2 - 24 \div 3 + 26$ (Answer: $49 - 8 + 26 = 67$)

(CA Mathematics Framework, adopted Nov. 6, 2013)

6.EE.1 Illustrative Task:
- Seven to the What?, https://www.illustrativemathematics.org/illustrations/891

a. What is the last digit of 7^{2011}? Explain.

b. What are the last *two* digits of 7^{2011}? Explain.

6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Essential Skills and Concepts:
- ☐ Write numerical expression involving whole number exponents
- ☐ Evaluate numerical expressions involving whole number exponents

Question Stems and Prompts:
- ✓ What do exponents represent?
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Vocabulary	**Spanish Cognates**
Tier 2	
• base	base
• powers	
Tier 3	
• numerical expression	expresión numérica
• exponents	exponente

Standards Connections
6.EE.1 → 6.EE.2

6.EE.1 Examples:

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6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.

6. EE. 1 Write and evaluate numerical expressions involving whole-number exponents.

Standard Explanation

Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal, or a positive fraction (6.EE.1▲). Students should work with a variety of expressions and problem situations to practice and deepen their skills. They can start with simple expressions to evaluate and move to more complex expressions. For example, they begin with simple whole numbers and move to fractions and decimal numbers. (MP.2, MP.6)

This is a foundational year for building the bridge between concrete concepts of arithmetic and the abstract thinking of algebra. Visual representations and concrete models (such as algebra tiles, counters, and cubes) can help students translate between concrete numerical representations and abstract symbolic representations.

Common Misconceptions: Students may not understand how to read the operations referenced with notations (e.g., x^3, $4x$, $3(x + 2y)$, $a + 3a$). Students are learning that

- x^3 means $x \cdot x \cdot x$, not $3x$ or 3 times x
- $4x$ means 4 times x or $x + x + x + x$, not forty-something
- When evaluating $4x$ when $x = 7$, substitution does not result in the expression meaning 47.
- For expressions like $a + 3a$, students need to see a as $1a$ to know that $a + 3a = 4a$ and not $3a^2$.

The use of the "x" notation as both the variable and the operation of multiplication can also be a source of confusion for students. In addition, students may need an explanation for why $x^0 = 1$ for all non-zero numbers x. Full explanations of this and other rules of working with exponents appear in grade eight.

(Adapted from Arizona 2012, N. Carolina 2012, and KATM 6th FlipBook 2012)

6.EE.2 Illustrative Task:

- Distance to School, https://www.illustrativemathematics.org/illustrations/540

Some of the students at Kahlo Middle School like to ride their bikes to and from school. They always ride unless it rains.

Let d be the distance in miles from a student's home to the school. Write two different expressions that represent how far a student travels by bike in a four week period if there is one rainy day each week.

6th Grade – CCSS for Mathematics

6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.

6. EE. 1 Write and evaluate numerical expressions involving whole-number exponents.

Standard Explanation

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6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* $5 - y$.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6 s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.

Essential Skills and Concepts:

- ☐ Write expressions involving numbers and letters
- ☐ Identify the parts of an expression
- ☐ Evaluate expressions using the order of operations
- ☐ Evaluate expressions that arise from real world formulas

Question Stems and Prompts:

- ✓ What are the different parts of an expression?
- ✓ How do you know when an expression is simplified?
- ✓ Why is it important to evaluate an expression in the correct order (i.e. using the order of operations)?
- ✓ How do you evaluate expressions? Explain your thinking for the given expression.

Math Vocabulary	**Spanish Cognates**
Tier 2	
• expression	expresión
• product	producto
• factor	factor
Tier 3	
• order of operations	open de operaciones
• sum	suma
• term	
• quotient	cociente
• coefficient	coeficiente

Standards Connections

6.EE.1 → 6.EE.3, 4, 6.EE.5, 6.EE.6

6th Grade – CCSS for Mathematics

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Math Vocabulary	**Spanish Cognates**
Tier 2	
• expression	expresión
• product	producto
• factor	factor
Tier 3	
• order of operations	open de operaciones
• sum	suma
• term	
• quotient	cociente
• coefficient	coeficiente

Standards Connections

6.EE.1 → 6.EE.3, 4, 6.EE.5, 6.EE.6

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6^{th} Grade – CCSS for Mathematics

6.EE.A2

Standard Explanation

Students write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2▲). Grade six students write expressions that record operations with numbers and with letters standing for numbers. Students need opportunities to read algebraic expressions to reinforce that the variable represents a number, and so behaves according to the same rules for operations as numbers do (e.g. distributive property).

Students identify the parts of an algebraic expression using mathematical terms such as variable, coefficient, constant, term, factor, sum, difference, product, and quotient. They should understand terms are the parts of a sum and when a term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable. Variables are letters that represent numbers. Development of this common language helps students understand the structure of expressions and explain their process for simplifying expressions. As students move from numerical to algebraic work, the multiplication and division symbols and are replaced by the conventions of algebraic notation. Students learn to use either a dot for multiplication, e.g., $\mathbf{1 \cdot 2 \cdot 3}$ instead of $\mathbf{1 \times 2 \times 3}$, or simple juxtaposition, e.g., $\boldsymbol{3x}$ instead of $\boldsymbol{3 \times x}$, which is potentially confusing (during the transition, students may indicate all multiplications with a dot, writing $\boldsymbol{3 \cdot x}$ for $\boldsymbol{3x}$).
Students also learn that can be written as $\boldsymbol{x \div 2}$ can be written as $\boldsymbol{\frac{x}{2}}$. (Adapted from Progressions 6-8 EE 2011.)

Examples of Expression Language. In the expression $x^2 + 5y + 3x + 6$,

- The variables are x and y.

- There are 4 terms, x^2, $5y$, $3x$, and 6.
- There are 3 variable terms, x^2, $5y$, $3x$. They have coefficients of 1, 5, and 3 respectively.
- The coefficient of x^2 is 1, since $x^2 = 1 \cdot x^2$.
- The term $5y$ represents $y + y + y + y + y$ or $5 \cdot y$.
- There is one constant term, 6.
- The expression shows a sum of all four terms.

Sixth grade students evaluate various expressions at specific values of their variables, including expressions that arise from formulas used in real-world problems. Examples where students evaluate the same expression at several different values of a variable are important for the later development of the concept of a function, and these should be experienced more frequently than problems wherein the values of the variables stay the same and the expression continues to change. (MP.1, MP.2, MP.3, MP.4, MP.6)

6.EE.A2

Standard Explanation

Students write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2▲). Grade six students write expressions that record operations with numbers and with letters standing for numbers. Students need opportunities to read algebraic expressions to reinforce that the variable represents a number, and so behaves according to the same rules for operations as numbers do (e.g. distributive property).

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6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.

6. EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Essential Skills and Concepts:
- ☐ Understand when two expressions are equivalent
- ☐ Create equivalent expressions
- ☐ Explain equivalent expressions using the properties of operations

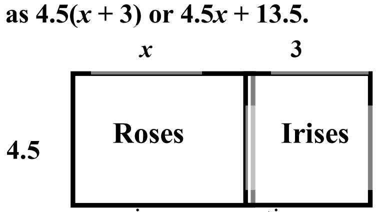
Question Stems and Prompts:
- ✓ How can you create equivalent expressions?
- ✓ Which properties of operations did you use to rewrite this expression in an equivalent form?
- ✓ How does understanding the properties help to create equivalent expressions?

Vocabulary	**Spanish Cognates**
Tier 2	
• expression	espresión
Tier 3	
• properties of operations	propiedades de operaciones
• distributive property	propiedad distributiva
• equivalent	equivalente
• algebra tiles	

Standards Connections
6.EE.3 → 7.EE.1

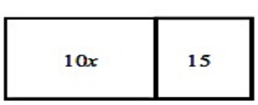
6.EE.3 Examples:

Given that the width is 4.5 units and the length can be represented by x + 3, the area of the flowers below can be expressed as 4.5(*x* + 3) or 4.5*x* + 13.5.



When given an expression representing area, students need to find the factors.

The expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x + 3).



(North Carolina Unpacking Document, July 2013)

6th Grade – CCSS for Mathematics

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Vocabulary	**Spanish Cognates**
Tier 2	
• expression	espresión
Tier 3	
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Standards Connections
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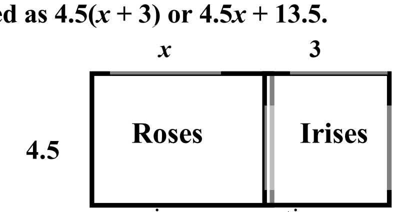

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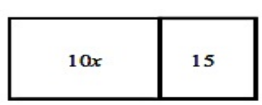

(North Carolina Unpacking Document, July 2013)

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6.EE.A.3

Standard Explanation

Students use their understanding of multiplication to interpret *3 (2 + x)* as 3 groups of *(2 + x)* or the area of a rectangle of lengths 3 units and *(2 + x)* units. (MP.2, MP.3, MP.4, MP.6, MP.7) They use a model to represent *x* and make an array to show the meaning of *3 (2 + x)*. They can explain why it makes sense *3 (2 + x)* that is equal to *6 + 3x*. Manipulatives such as “algebra tiles,” which make use of the area model to represent quantities, can be used to show why this is true. Note that with algebra tiles a 1 x 1 square represents a unit (the number 1), while the variable *x* is represented by a rectangle of 1 x *x* dimensions (that is, the longer side of the *x* - tile is not commensurate with a whole number of unit tiles, and therefore represents an “unknown” length).

Important in standards (6.EE.3▲) and (6.EE.4▲) is for students to understand that the distributive property is the basis for combining “like” terms in an expression (or equation). For instance, students understand that $\boldsymbol{4a + 7a = 11a}$, because $\boldsymbol{4a + 7a = (4+7)a = 11a}$.

This ability to use the distributive property “forwards” and “backwards” is important for students to develop. Students generate equivalent expressions in general using the associative, commutative, and distributive properties and can prove the expressions are equivalent (MP.1, MP.2, MP.3, MP.4, MP.6). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

EE Progression Information:

As students move from numerical to algebraic work the multiplication and division symbols × and ÷ are replaced by the conventions of algebraic notation. Students learn to use either a dot for multiplication, e.g., $\boldsymbol{1 \cdot 2 \cdot 3}$ instead of $\boldsymbol{1 \times 2 \times 3}$, or simple juxtaposition, e.g., $\boldsymbol{3x}$ instead of $\boldsymbol{3 \times x}$ (during the transition, students may indicate all multiplications with a dot, writing $\boldsymbol{3 \cdot x}$ for $\boldsymbol{3x}$). A firm grasp on variables as numbers helps students extend their work with the properties of operations from arithmetic to algebra. For example, students who are accustomed to mentally calculating $\boldsymbol{5 \times 37}$ as $\boldsymbol{5 \times (30+7) = 150 + 35}$ can now see that $\boldsymbol{5\,(3a+7) = 15a + 35}$ for all numbers $\boldsymbol{a}$.

6.EE.3 Illustrative Task:

- Anna in D.C., https://www.illustrativemathematics.org/illustrations/997

Anna enjoys dinner at a restaurant in Washington, D.C., where the sales tax on meals is 10%. She leaves a 15% tip on the price of her meal before the sales tax is added, and the tax is calculated on the pre-tip amount. She spends a total of $27.50 for dinner. What is the cost of her dinner without tax or tip?

6th Grade – CCSS for Mathematics

6.EE.A.3

Standard Explanation

Students use their understanding of multiplication to interpret *3 (2 + x)* as 3 groups of *(2 + x)* or the area of a rectangle of lengths 3 units and *(2 + x)* units. (MP.2, MP.3, MP.4, MP.6, MP.7) They use a model to represent *x* and make an array to show the meaning of *3 (2 + x)*. They can explain why it makes sense *3 (2 + x)* that is equal to *6 + 3x*. Manipulatives such as “algebra tiles,” which make use of the area model to represent quantities, can be used to show why this is true. Note that with algebra tiles a 1 x 1 square represents a unit (the number 1), while the variable *x* is represented by a rectangle of 1 x *x* dimensions (that is, the longer side of the *x* - tile is not commensurate with a whole number of unit tiles, and therefore represents an “unknown” length).

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6th Grade – CCSS for Mathematics

6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE. 4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* $y + y + y$ *and* $3y$ *are equivalent because they name the same number regardless of which number* y *stands for.*

Essential Skills and Concepts:

- ☐ Identify equivalent expressions
- ☐ Explain why expressions are equivalent
- ☐ Understand that two expressions are equivalent because they name the same value regardless of the number substituted for the variable

Question Stems and Prompts:

- ✓ Identify equivalent expressions and explain why they are equivalent.
- ✓ Demonstrate that two expressions are equivalent by substituting values in for the variable.
- ✓ Identify expressions that are not equivalent to the given expression and explain why they are not equivalent.

Vocabulary	**Spanish Cognates**
Tier 2	
• substitute	sustituto
Tier 3	
• equivalent expressions	espresión equivalente
• algebra tiles	

Standards Connections

6.EE.4 → 7.EE.1

6.EE.4 Examples:

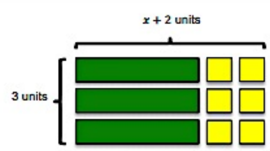
Are the expressions equivalent? Explain your answer?

$4m + 8$, $4(m+2)$, $3m + 8 + m$, $2 + 2m + m + 6 + m$

Expression	Simplifying the Expression	Explanation
$4m + 8$	$4m + 8$	Already in simplest form
$4(m+2)$	$4(m+2)$ $4m + 8$	*Distributive property*
$3m + 8 + m$	$3m + 8 + m$ $3m + m + 8$ $4m + 8$	*Combined like terms*
$2 + 2m + m + 6 + m$	$2m + m + m + 2 + 6$ $4m + 8$	*Combined like terms*

(North Carolina Unpacking Doc., July 2013) (CA Mathematics Framework, 2013)

Example of Basic Reasoning with Algebra Tiles:
Students can recognize $3(x + 2)$ as representing the area of a rectangle of lengths 3 units and $(x + 2)$ units. Using the appropriate number of tiles (or a sketch), students can see that there are $3 \cdot 2 = 6$ and $3 \cdot x = 3x$ units altogether, so that $3(x + 2) = 3x + 6$.



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Vocabulary	**Spanish Cognates**
Tier 2	
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Standards Connections

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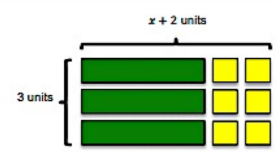

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6.EE.A.4

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Example: Equivalent Expressions.
Show that the two expressions $5(n+3)+7n$ and $12n+15$ are equivalent. **Solution:** By applying the distributive property, I know that $5(n+3)+7n$ can be rewritten as $5n+15+7n$ because of the distributive property. Also, since $5n+7n=(5+7)n=12n$, I can write the expression as $12n+15$.

(Adapted from Arizona 2012, N. Carolina 2012, and KATM FlipBook 2012)

6.EE.4 Illustrative Task:

- Equivalent Expressions, https://www.illustrativemathematics.org/illustrations/542

Which of the following expressions are equivalent? Why? If an expression has no match, write 2 equivalent expressions to match it.

a. $2(x+4)$

b. $8+2x$

c. $2x+4$

d. $3(x+4)-(4+x)$

e. $x+4$

6^{th} Grade – CCSS for Mathematics

6.EE.A.4

Standard Explanation

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6.EE.B. Reason about and solve one-variable equations and inequalities.

6.EE. 5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Standard Explanation

In elementary grades, students explored the concept of equality. In 6th grade, students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true. (North Carolina Unpacking Document, July 2013)

EE Progression Information:

In Grades K–5 students have been writing numerical equations and simple equations involving one operation with a variable. In Grade 6 they start the systematic study of equations and inequalities and methods of solving them. Solving is a process of reasoning to find the numbers which make an equation true, which can include checking if a given number is a solution. Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off, such as in $\boldsymbol{4x+3x=3x+20}$, where they can see that 4x must be 20 to make the two sides equal.

6th Grade – CCSS for Mathematics

6.EE.B. Reason about and solve one-variable equations and inequalities.

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6.EE.B Reason about and solve one-variable equations and inequalities.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Essential Skills and Concepts:

- ☐ Use variables to represent numbers and write expressions
- ☐ Write expressions to represent real world problems
- ☐ Understand that variables can represent an unknown number

Question Stems and Prompts:

- ✓ What do variables represent?
- ✓ Write a real-world situation to represent the expression.
- ✓ Explain how the expression represents the real world situation.

Vocabulary	**Spanish Cognates**
Tier 3	
• variable	variable
• expression	expresión
• algebra tiles	

Standards Connections

6.EE.6 – 6.EE.7

6.EE.6 Illustrative Tasks:

- Firefighter Allocation, https://www.illustrativemathematics.org/illustrations/425

A town's total allocation for firefighter's wages and benefits in a new budget is $600,000. If wages are calculated at $40,000 per firefighter and benefits at $20,000 per firefighter, write an equation whose solution is the number of firefighters the town can employ if they spend their whole budget. Solve the equation.

- Pennies to Heaven, https://www.illustrativemathematics.org/illustrations/1291

A penny is about $\frac{1}{16}$ of an inch thick.

a. In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

b. In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

c. The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?

6th Grade – CCSS for Mathematics

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Vocabulary / **Spanish Cognates**
Tier 3
• variable — variable
• expression — expresión
• algebra tiles

Standards Connections

6.EE.6 – 6.EE.7

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6th Grade – CCSS for Mathematics

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Standard Explanation

In the elementary grades students explored the concept of equality. In sixth grade students explore equations as one expression being set equal to a specific value. A solution is a value of the variable that makes the equation true. Students use various processes to identify such value(s) that when substituted for the variable will make the equation true (6.EE.5▲). Students can use manipulatives and pictures (e.g., tape-like diagrams) to represent equations and their solution strategies. When writing equations, students learn to be precise in their definition of a variable, e.g., writing “ equals John’s age in years” as opposed to simply writing “ is John.” (6.EE.6▲). (MP.6) (CA *Mathematics Framework*, adopted Nov. 6, 2013)

EE Progression Information:

As with all their work with variables, it is important for students to state precisely the meaning of variables they use when setting up equations (MP6). This includes specifying whether the variable refers to a specific number, or to all numbers in some range. For example, in the equation $\mathbf{0.44n = 11}$ the variable $\boldsymbol{n}$ refers to a specific number (the number of stamps you can buy for \$11); however, if the expression $\mathbf{0.44n}$ is presented as a general formula for calculating the price in dollars of $\boldsymbol{n}$ stamps, then $\mathbf{n}$ refers to all numbers in some domain. That domain might be specified by inequalities, such as $\boldsymbol{n > 0}$. [6.EE.8]

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In the elementary grades students explored the concept of equality. In sixth grade students explore equations as one expression being set equal to a specific value. A solution is a value of the variable that makes the equation true. Students use various processes to identify such value(s) that when substituted for the variable will make the equation true (6.EE.5▲). Students can use manipulatives and pictures (e.g., tape-like diagrams) to represent equations and their solution strategies. When writing equations, students learn to be precise in their definition of a variable, e.g., writing “ equals John’s age in years” as opposed to simply writing “ is John.” (6.EE.6▲). (MP.6) (CA *Mathematics Framework*, adopted Nov. 6, 2013)

EE Progression Information:

As with all their work with variables, it is important for students to state precisely the meaning of variables they use when setting up equations (MP6). This includes specifying whether the variable refers to a specific number, or to all numbers in some range. For example, in the equation $\mathbf{0.44n = 11}$ the variable $\boldsymbol{n}$ refers to a specific number (the number of stamps you can buy for \$11); however, if the expression $\mathbf{0.44n}$ is presented as a general formula for calculating the price in dollars of $\boldsymbol{n}$ stamps, then $\mathbf{n}$ refers to all numbers in some domain. That domain might be specified by inequalities, such as $\boldsymbol{n > 0}$. [6.EE.8]

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6.EE.B Reason about and solve one-variable equations and inequalities.

6. EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Essential Skills and Concepts:

- ☐ Write one variable equations and inequalities for real-world and mathematical problems
- ☐ Solve one variable equations
- ☐ Solve one variable inequalities

Question Stems and Prompts:

- ✓ Explain how you wrote the equation for the given situation.
- ✓ Solve the equation/inequality. Explain your thinking process.
- ✓ How are solving equations and inequalities related?

Vocabulary	Spanish Cognates
Tier 2	
• variable	variable
Tier 3	
• equation	ecuación
• inequality	desigualdad

Standards Connections

6.EE.5 → 6.EE.3, 4

6.EE.7 Illustrative Tasks:

- Morning Walk, https://www.illustrativemathematics.org/illustrations/1107

Sierra walks her dog Pepper twice a day. Her evening walk is two and a half times as far as her morning walk. At the end of the week she tells her mom,

I walked Pepper for 30 miles this week!

How long is her morning walk?

- Fruit Stand, https://www.illustrativemathematics.org/illustrations/1032

A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries.

How many cherries are there in the fruit salad?

6th Grade – CCSS for Mathematics

6.EE.B Reason about and solve one-variable equations and inequalities.

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6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Standard Explanation

Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer; however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.

Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include fractions and decimals with non-negative solutions. Students recognize that dividing by 6 and multiplying by $\frac{1}{6}$ produces the same result.

For example, $\frac{x}{6} = 9$ and $\frac{1}{6}x = 9$ will produce the same result.

Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem. (North Carolina Unpacking Document, July 2013)

6.EE.7 Examples:

Examples: Solving Equations of the Form $p + x = q$ and $px = q$. (6.EE.7▲).

1. Joey had 26 game cards. His friend Richard gave him some more and now he has 100 cards. How many cards did his friend Richard give him? Write and equation and solve your equation.
   Solution: Since Richard gave him some more cards, we let n be the number of cards that Richard gave Joey. This means he now has $26 + n$ cards. But the number of cards Joey has is 100, so we get the equation $26 + n = 100$. Using the relationship between addition and subtraction, we see that $n = 100 - 26 = 74$, which means that his friend gave him 74 cards. One can represent this equation with a tape-like diagram:

100	
26	n

3. Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an equation that represents this situation and solve it to determine how much one pair of jeans cost.
   Solution: If J represents the cost of one pair of jeans in dollars, then the equation becomes $3J = 56.58$. If we solve this for J, we find $J = 56.58 \div 3 = 18.86$. This means each pair of jeans cost $18.86.

$56.58		
J	J	J

4. Julio gets paid $20 for babysitting. He spent $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julio has left.
   Solution: One equation might be $1.99 + 6.50 + x = 20.00$, where x represents how many dollars he has left. We find that $x = 11.51$, so that he has $11.51 left.

(Adapted from Arizona 2012, N. Carolina 2012, and KATM FlipBook 2012)

6th Grade – CCSS for Mathematics

6.EE.B Reason about and solve one-variable equations and inequalities.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Standard Explanation

Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known. For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer; however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. Problems should be in context when possible and use only one variable.

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6.EE.B Reason about and solve one-variable equations and inequalities.

6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Essential Skills and Concepts:
- ☐ Represent numbers using variables for equations and inequalities
- ☐ Write expressions to solve real-world problems
- ☐ Understand that variables can represent any number

Question Stems and Prompts:
- ✓ What do variables represent?
- ✓ Write an expression to represent a real-world problem. Explain why you wrote your expression the way you did.

Vocabulary	**Spanish Cognates**
Tier 2	
• variable	variable
• expression	expresión
• set	
• unknown	

6.EE.8 Illustrative Tasks:
- Fishing Adventures 1, https://www.illustrativemathematics.org/illustrations/642

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 900 pounds of weight for safety reasons.

a. Let *p* represent the total number of people. Write an inequality to describe the number of people that a boat can hold. Draw a number line diagram that shows all possible solutions.

b. Let *w* represent the total weight of a group of people wishing to rent a boat. Write an inequality that describes all total weights allowed in a boat. Draw a number line diagram that shows all possible solutions.

6th Grade – CCSS for Mathematics

6.EE.B Reason about and solve one-variable equations and inequalities.

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6.EE.B Reason about and solve one-variable equations and inequalities.

6.EE.8 Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

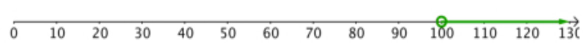
Standard Explanation

Many real-world situations are represented by inequalities. In grade six, students write simple inequalities involving or to represent real world and mathematical situations, and they use the number line to model the solutions (6.EE.8▲). Students learn that when representing inequalities of these forms on a number line, the common practice is to draw an arrow on or above the number line with an open circle on or above the number in the inequality. The arrow indicates the numbers greater than or less than the number in question, and that the solutions extend indefinitely. The arrow is a solid line, to indicate that even fractional and decimal amounts (i.e. points between dashes on the are included in the solution set). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Examples: Inequalities of the Form $x < c$ and $x > c$.

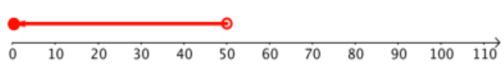
1. The class must raise more than $100 to go on the field trip. Let m represent the amount of money in dollars that the class raises. Write an inequality that describes how much the class needs to raise. Represent this on a number line.

   Solution: Since the amount of money, m, needs to be greater than 100, the inequality is $m > 100$. A number line diagram for this might look like:



2. The Flores family spent less than $50.00 on groceries last week. Write an inequality that describes this situation and graph the solution on a number line.

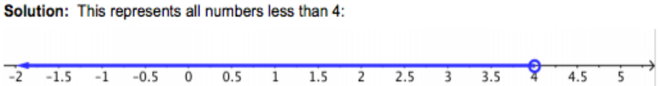
   Solution: If we let g represent the amount of money in dollars the family spent on groceries last week, then the inequality becomes $k < 50$. We might represent this in the following way:



   (In this example, it doesn't make sense that the Flores family could have spent a negative amount of dollars on groceries, so the arrow would stop precisely at $0; we would typically represent this with a dot over 0 rather than the arrow.)

3. Graph $x < 4$.

   Solution: This represents all numbers less than 4:



(Adapted from Arizona 2012, N. Carolina 2012, and KATM FlipBook 2012)

6th Grade – CCSS for Mathematics

6.EE.B Reason about and solve one-variable equations and inequalities.

6.EE.8 Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

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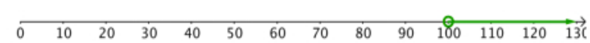

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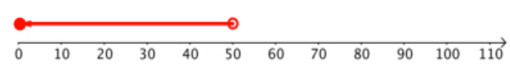

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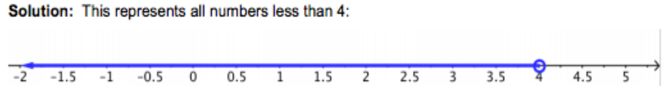

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6th Grade – CCSS for Mathematics

6.EE.C Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Essential Skills and Concepts:
- ☐ Use variables to represent two related quantities
- ☐ Write an equation to express one quantity (dependent variable) in terms of the other (independent variable)
- ☐ Analyze the dependent and independent relationship between variables

Question Stems and Prompts:
- ✓ What is the relationship between dependent and independent variables?
- ✓ How do graphs and tables show the relationship between dependent and independent variables?

Vocabulary	**Spanish Cognates**
Tier 2	
• variable	variable
Tier 3	
• independent variable	variable independiente
• dependent variable	variable dependiente

Standards Connections
6.EE.9 – 6.EE.7, 6.RP.3b

6.EE.9 Illustrative Task:
- Chocolate Bar Sales, https://www.illustrativemathematics.org/illustrations/806

Stephanie is helping her band collect money to fund a field trip. The band decided to sell boxes of chocolate bars. Each bar sells for $1.50 and each box contains 20 bars. Below is a partial table of monies collected for different numbers of boxes sold.

Boxes Sold *b*	Money Collected *m*
1	$30.00
2	
3	
4	
5	$150.00
6	
7	
8	

a. Complete the table above for values of *m*.

b. Write an equation for the amount of money, *m*, that will be collected if *b* boxes of chocolate bars are sold. Which is the independent variable and which is the dependent variable?

c. Graph the equation using the ordered pairs from the table above.

d. Calculate how much money will be collected if 100 boxes of chocolate bars are sold.

e. The band collected $1530.00 from chocolate bar sales. How many boxes did they sell?

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Standard Explanation

In grade six students investigate the relationship between two variables, beginning with the distinction between dependent and independent variables (6.EE.9▲). The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the -axis; the dependent variable is graphed on the -axis. They also understand that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only.

Students represent relationships between quantities with multiple representations, including describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the relationship. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

EE Progression Information:

In addition to constructing and solving equations in one variable, students use equations in two variables to express relationships between two quantities that vary together. When they construct an expression like $10 - p$ to represent a quantity, students can choose a variable such as C to represent the calculated quantity and write $C = 10 - p$ to represent the relationship. This prepares students for work with functions in later grades. The variable p is the natural choice for the independent variable in this situation, with C the dependent variable. In a situation where the price, p, is to be calculated from the change, C, it might be the other way around.

As they work with such equations students begin to develop a dynamic understanding of variables, an appreciation that they can stand for any number from some domain. This use of variables arises when students study expressions such as $0.44n$, discussed earlier, or equations in two variables such as $d = 5 + 5t$ describing the relationship between distance in miles, d, and time in hours, t.

6th Grade – CCSS for Mathematics

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6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Essential Skills and Concepts:
- ☐ Find the area of right triangles, other triangles, special quadrilaterals, and polygons
- ☐ Compose and decompose shapes into rectangles and triangles to find their area
- ☐ Solve real-world problems involving area by composing and decomposing shapes

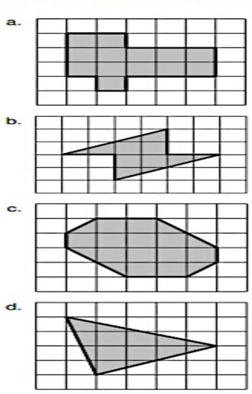
Question Stems and Prompts:
- ✓ What is the relationship between triangles and quadrilaterals?
- ✓ Why is it useful to compose and decompose shapes to find area?
- ✓ How is the area used in real-world situations?

Math Vocabulary	**Spanish Cognates**
Tier 2	
• compose	componer
• decompose	descomponer
Tier 3	
• triangles	triángulo
• quadrilateral	cuadrilátero
• polygon	polígono

Standards Connections
6.G.1 → 6.G.4

6.G.1 Illustrative Task:
- Finding Areas of Polygons, https://www.illustrativemathematics.org/illustrations/647

Find the area that is shaded in each figure in at least two different ways.



6^{th} Grade – CCSS for Mathematics

6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Essential Skills and Concepts:
- ☐ Find the area of right triangles, other triangles, special quadrilaterals, and polygons
- ☐ Compose and decompose shapes into rectangles and triangles to find their area
- ☐ Solve real-world problems involving area by composing and decomposing shapes

Question Stems and Prompts:
- ✓ What is the relationship between triangles and quadrilaterals?
- ✓ Why is it useful to compose and decompose shapes to find area?
- ✓ How is the area used in real-world situations?

Math Vocabulary	**Spanish Cognates**
Tier 2	
• compose	componer
• decompose	descomponer
Tier 3	
• triangles	triángulo
• quadrilateral	cuadrilátero
• polygon	polígono

Standards Connections
6.G.1 → 6.G.4

6.G.1 Illustrative Task:
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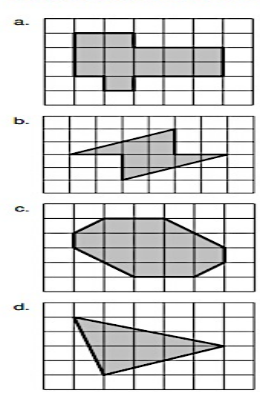

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6.G.A.1

Standard Explanation

In grade six students extend their understanding of length, area, and volume as they solve problems by applying formulas for the area of triangles and parallelograms and volume of rectangular prisms.

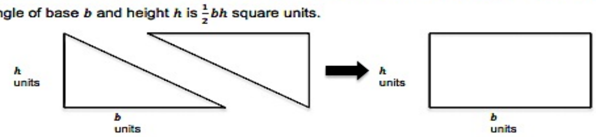
Students in grade six build on their work with area in earlier grades by reasoning about relationships among shapes to determine area, surface area, and volume. Students continue to understand area as the number of squares needed to cover a plane figure. Sixth grade students find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. As students compose and decompose shapes to determine areas, they learn that area is conserved when composing or decomposing shapes. For example, students will decompose trapezoids into triangles and/or rectangles, and use this reasoning to find formulas for the area of a trapezoid. Students know area formulas for triangles and some special quadrilaterals, in the sense of having an understanding of why the formula works and how the formula relates to the measure (area) and the figure (6.G.1).

Prior to seeing formulas for areas of different shapes, students can find areas of shapes on centimeter grid paper by duplicating, composing, and decomposing shapes. These experiences will make them familiar with the processes that result in the derivations of the formulas shown below.

Deriving Area Formulas (MP.3, MP.7).

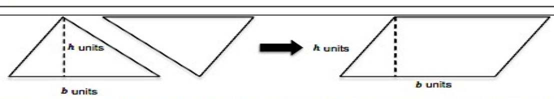
Starting with a basic understanding of the area of a rectangle of base b units and height h units being bh square units, along with the relationship between rectangles and triangles, and the law of conservation of area, students can justify area formulas for various shapes.

Right Triangles: Since two right triangles of base b and height h can be composed to form a rectangle of the same base and height, the triangle must have an area ½ that of the rectangle. Thus, the area of a right triangle of base b and height h is $\frac{1}{2}bh$ square units.

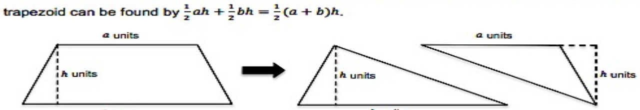


Parallelograms: If we define the height of the parallelogram to be the length of a perpendicular segment from base to base, then a parallelogram of base b and height h has the same area (bh square units) as a rectangle of the same dimensions. We cut off a right triangle as shown, and move it to complete the rectangle.

Non-Right Triangles: Non-right triangles of base b units and height h units can now be duplicated to make parallelograms. By similar reasoning used with right triangles and rectangles, the area of such a triangle is $\frac{1}{2}bh$ square units. (One can show the same holds true for obtuse triangles.)



Trapezoids: Trapezoids can be deconstructed into two triangles of bases a and b, showing that the area of a trapezoid can be found by $\frac{1}{2}ah + \frac{1}{2}bh = \frac{1}{2}(a+b)h$.



6th Grade – CCSS for Mathematics

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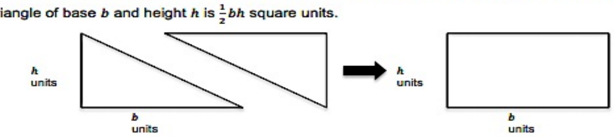

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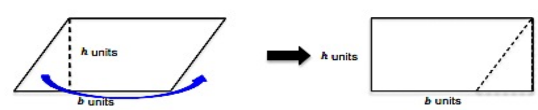

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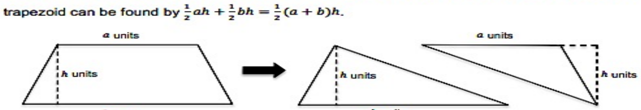

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6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Essential Skills and Concepts:
- ☐ Find the volume of a right rectangular prism with fractional edge lengths
- ☐ Use unit cubes to find the volume
- ☐ Use the formula for volume to solve real-world problems
- ☐ Demonstrate that the volume is the same whether packing the prism with unit cubes or calculating using the formula

Question Stems and Prompts:
- ✓ How can the volume be found using unit cubes?
- ✓ Compare finding the volume using unit cubes to calculating the volume using the formula.
- ✓ Give examples of finding the volume of prisms in real-word situations. Apply the volume formula and explain your thought process.

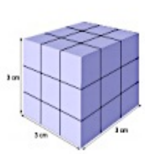
Math Vocabulary	**Spanish Cognates**
Tier 3	
• volume	volumen
• length	
• cube	cubo
• prism	prisma

6.G.2 Illustrative Tasks:
- Banana Bread, https://www.illustrativemathematics.org/illustrations/657

Leo's recipe for banana bread won't fit in his favorite pan. The batter fills the 8.5 inch by 11 inch by 1.75 inch pan to the very top, but when it bakes it spills over the side. He has another pan that is 9 inches by 9 inches by 3 inches, and from past experience he thinks he needs about an inch between the top of the batter and the rim of the pan. Should he use this pan?

- Computing Volume Progression 1, https://www.illustrativemathematics.org/illustrations/534

a. Amy wants to build a cube with 3 *cm* sides using 1 *cm* cubes. How many cubes does she need?



b. How many 1 *cm* cubes would she need to build a cube with 6 *cm* sides?

6th Grade – CCSS for Mathematics

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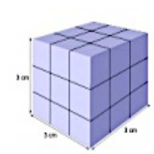

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6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

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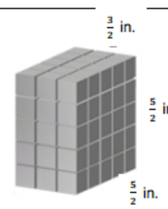
Standard Explanation

Previously, students calculated the volume of right rectangular prisms using whole number edges, and understood doing so as finding the number of unit cubes (i.e. cubic unit) within a solid shape. In grade six students extend this work to unit cubes with fractional edge lengths, e.g., they find volumes by finding the number of units cubes of dimensions within a figure with fractional side lengths. Students draw diagrams to represent fractional side lengths, connecting finding these volumes with multiplication of fractions. (6.G.2)

Finding areas, surface areas and volumes present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6). (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Example: Counting Fractional Cubic Units.

The model shows a rectangular prism with dimensions $\frac{3}{2}$ inches, $\frac{5}{2}$ inches, and $\frac{5}{2}$ inches. Each of the cubic units shown in the model has a volume of $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ cubic inches. Students should reason why each of these units has this volume (i.e. by discovering that 8 of them fit in a $1 \times 1 \times 1$-cube). Furthermore, students explain why the volume of the rectangular prism is given by $\frac{3}{2} \times \frac{5}{2} \times \frac{5}{2}$ cubic inches, and why one can also find the volume by finding $(3 \times 5 \times 5) \times \left(\frac{1}{8} \text{ cubic inch}\right)$.



(Adapted from Arizona 2012)

6th Grade – CCSS for Mathematics

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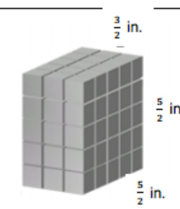

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6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Essential Skills and Concepts:
- ☐ Draw polygons using coordinates in a coordinate plane
- ☐ Find the lengths of the sides using the coordinates
- ☐ Solve real-world problems involving polygons in the coordinate plane

Question Stems and Prompts:
- ✓ How can you find the length of the sides using the coordinates for the vertices and plotting them?
- ✓ Describe how you plotted and connected the vertices to create a polygon in the coordinate plane.

Math Vocabulary	**Spanish Cognates**
Tier 3	
• polygon	polígono
• length	
• coordinate plane	plano de coordenadas
• vertex/vertices	vertice

Standards Connections
6.G.3 – 6.NS.8

6.G.3 Illustrative Task:
- Polygons in the Coordinate Plane, https://www.illustrativemathematics.org/illustrations/1188

The vertices of eight polygons are given below. For each polygon:

- Plot the points in the coordinate plane connect the points in the order that they are listed.
- Color the shape the indicated color and identify the type of polygon it is.
- Find the area.

a. The first polygon is GREY and has these vertices:

$(-7,4)\ (-8,5)\ (-8,6)\ (-7,7)\ (-5,7)\ (-5,5)\ (-7,4)$

b. The second polygon is ORANGE and has these vertices:

$(-2,-7)\ (-1,-4)\ (3,-1)\ (6,-7)\ (-2,-7)$

c. The third polygon is GREEN and has these vertices:

$(4,3)\ (3,3)\ (2,2)\ (2,1)\ (3,0)\ (4,0)\ (5,1)\ (5,2)\ (4,3)$

d. The fourth polygon is BROWN and has these vertices:

$(0,-10)\ (0,-8)\ (7,-10)\ (0,-10)$

6th Grade – CCSS for Mathematics

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Standard Explanation

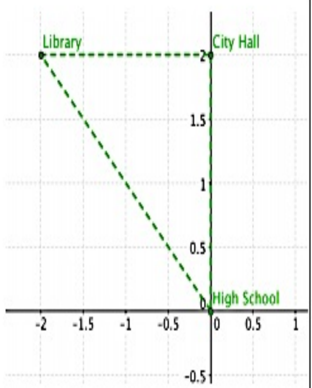
Finding areas, surface areas and volumes present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6).

In standard 6.G.3, students represent shapes in the coordinate plane. They find lengths of sides that contain vertices with a common - or -coordinate, representing an important step for later grade eight understanding of how to use the distance formula to find the distance between any two points in the plane. In addition, in grade six students construct three-dimensional shapes using nets and build on their work with areas (6.G.4) by finding surface areas using nets. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Example: Polygons in the Coordinate Plane.

On a grid map, the library is located at (−2, 2), the city hall building is located at (0, 2), and the high school is located at (0, 0).

a. Represent the locations as points on a coordinate grid with a unit of 1 mile.
b. What is the distance from the library to the city hall building?
c. What is the distance from the city hall building to the high school? How do you know?
d. What is the shape that results from connecting the three locations with line segments?
e. The city council is planning to place a city park in this area. What is the area of the planned park?



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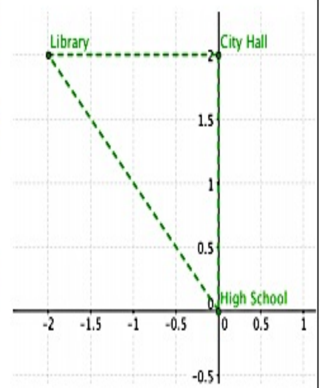

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6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Essential Skills and Concepts:

- ☐ Use nets to represent three-dimensional figures
- ☐ Use nets to find the surface area of the figures in real-world and mathematical problems

Question Stems and Prompts:

- ✓ How can nets be used to represent three-dimensional figures and model real-world problems?
- ✓ How can you use nets to find the surface area of three-dimensional figures?

Math Vocabulary	Spanish Cognates
Tier 2	
• figure	figura
• net	
Tier 3	
• three-dimensional	tridimensional
• rectangles	rectángulo
• triangles	triángulo
• surface area	área de superficie

6.G.4 Resource:

Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations: http://illuminations.nctm.org/ActivityDetail.aspx?ID=205.

6th Grade – CCSS for Mathematics

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Standard Explanation

Finding areas, surface areas and volumes present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6).

A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure.

Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area. (North Carolina Unpacking Document, July 2013)

K – 6, Geometry Progression Information:

Composition and decomposition of shapes is used throughout geometry from Grade 6 to high school and beyond. Compositions and decompositions of regions continues to be important for solving a wide variety of area problems, including justifications of formulas and solving real world problems that involve complex shapes. Decompositions are often indicated in geometric diagrams by an auxiliary line, and using the strategy of drawing an auxiliary line to solve a problem are part of looking for and making use of structure (MP7). Recognizing the significance of an existing line in a figure is also part of looking for and making use of structure. This may involve identifying the length of an associated line segment, which in turn may rely on students' abilities to identify relationships of line segments and angles in the figure. These abilities become more sophisticated as students gain more experience in geometry. In Grade 7, this experience includes making scale drawings of geometric figures and solving problems involving angle measure, surface area, and volume.

6th Grade – CCSS for Mathematics

6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Standard Explanation

Finding areas, surface areas and volumes present modeling opportunities (MP.4) and require students to attend to precision with the types of units involved (MP.6).

A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure.

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6th Grade – CCSS for Mathematics

6.SP.A Develop understanding of statistical variability.

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

Essential Skills and Concepts:
- ☐ Recognize statistical questions and explain what makes them statistical
- ☐ Create statistical questions

Question Stems and Prompts:
- ✓ What is a statistical question?
- ✓ How does a statistical question anticipate variability?
- ✓ Give examples of statistical questions.

Vocabulary	**Spanish Cognates**
Tier 2	
• variability	variabilidad
Tier 3	
• statistical questions	

Standards Connections
6.SP.1 → 6.SP.3

6.SP.1 Illustrative Tasks:
- Buttons: Statistical Questions, https://www.illustrativemathematics.org/illustrations/1040

a. Which of the following are statistical questions that someone could ask Zeke about his buttons? (A statistical question is one that anticipates an answer based on data that vary.) For each question, explain why it is or is not a statistical question.

i. What is a typical number of holes for the buttons in the jar?
ii. How many buttons are in the jar?
iii. How large is the largest button in the jar?
iv. If Zeke grabbed a handful of buttons, what are the chances that all of the buttons in his hand are round?
v. What is a typical size for the buttons in the jar?
vi. How are these buttons distributed according to color?

b. Write another statistical question related to Zeke's button collection

- Identifying Statistical Questions, https://www.illustrativemathematics.org/illustrations/703

Which of the following are statistical questions? (A statistical question is one that can be answered by collecting data and where there will be variability in that data.)

a. How many days are in March?
b. How old is your dog?
c. How old are the dogs on this street?
d. What percent of people like watermelons?
e. Do you like watermelons?
f. How many bricks are in this wall?
g. What was the highest temperature today in town?

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Vocabulary — **Spanish Cognates**
Tier 2
• variability — variabilidad
Tier 3
• statistical questions

Standards Connections
6.SP.1 → 6.SP.3

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6th Grade – CCSS for Mathematics

6.SP.A.1

Standard Explanation

A critical area of instruction in grade six is developing understanding of statistical thinking. Students build on their knowledge and experiences in data analysis as they work with statistical variability and represent and analyze data distributions. They continue to think statistically, viewing statistical reasoning as a four-step investigative process:

Four-Step Statistical Investigation.
• Formulate questions that can be answered with data. • Design and use a plan to collect relevant data. • Analyze the data with appropriate methods. • Interpret results and draw valid conclusions from the data that relate to the questions posed.

Statistical investigations start with questions, which can result in a narrow or wide range of numerical values, and ultimately result in some degree of variability (6.SP.1). For example, asking classmates "How old are the students in my class in years?" will result in less variability than asking "How old are the students in my class in months?" Students understand questions need to specifically demand measureable answers, e.g., if a student wants to know about the exercise habits of the students at their school, rather than asking "Do you exercise?" a statistical question for this study could be "How many hours per week on average do students at Jefferson Middle School exercise?" Grade six students design survey questions that anticipate variability. They understand surveys include a variety of possible responses with specific numerical answers (e.g., 3 hours per week, 4 hours per week). (Adapted from Arizona 2012, N. Carolina 2012, and KATM FlipBook 2012)

A major focus of grade six is the characterization of data distributions by measures of center and spread. To be useful, center and spread must have well-defined numerical descriptions that are commonly understood by those using the results of a statistical investigation (Progressions 6-8 SP 2011). Sixth grade students analyze the center, spread, and overall shape of a set of data (6.SP.1). As students analyze and/or compare data sets they consider the context in which the data is collected and identify clusters, peaks, gaps, and symmetry in the data. Students learn that data sets contain many numerical values that can be summarized by one number such as a measure of center (mean and median) and range. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Describing Data.
The *measure of center* gives a numerical value to represent the center of the data (e.g., midpoint of an ordered list or the balancing point). The *range* provides a single number that describes how the values vary across the data set. Another characteristic of a data set is the measure of *variability* (or spread) of the values.

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6.SP.A Develop understanding of statistical variability.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

Essential Skills and Concepts:
- ☐ Understand statistical variability
- ☐ Describe a data distribution based on its center, spread, and overall shape

Question Stems and Prompts:
- ✓ How can a data distribution be described?
- ✓ Describe the center of the distribution
- ✓ Describe the spread of the distribution
- ✓ Describe the overall shape of the distribution

Vocabulary	**Spanish Cognates**
Tier 2	
• center	centro
• spread	
• shape	
Tier 3	
• statistical question	
• data distribution	distribución de datos

Standards Connections
6.SP.2 → 6.SP.3, 5

6.SP.2, 5d Illustrative Tasks:
- Puppy Weights, https://www.illustrativemathematics.org/illustrations/1026

Below are the 25 birth weights, in ounces, of all the Labrador Retriever puppies born at Kingston Kennels in the last six months.

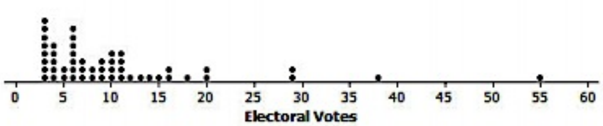
13 14 15 15 16 16 16 16 17 17 17 17 17 17 17 18 18 18 18 18 18 18 18 19 20

a. Use an appropriate graph to summarize these birth weights.

b. Describe the distribution of birth weights for puppies born at Kingston Kennels in the last six months. Be sure to describe shape, center and variability.

c. What is a typical birth weight for puppies born at Kingston Kennels in the last six months? Explain why you chose this value.

- Electoral College, https://www.illustrativemathematics.org/illustrations/1199



6th Grade – CCSS for Mathematics

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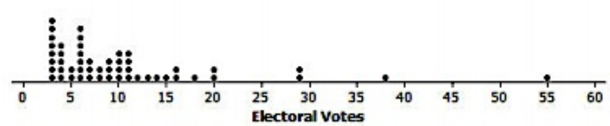

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6.SP.A Develop understanding of statistical variability.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

Standard Explanation

The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots. (North Carolina Unpacking Document, July 2013)

6th Grade – CCSS for Mathematics

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6.SP.A Develop understanding of statistical variability.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Essential Skills and Concepts:
- ☐ Understand and explain measures of center
- ☐ Understand and explain measures of variation

Question Stems and Prompts:
- ✓ What is the difference between measures of center and a measure of variation?
- ✓ Describe a data set in terms of measures of center.
- ✓ Describe a data set in terms of measures of variability.

Vocabulary	**Spanish Cognates**
Tier 2	
• value	valor
Tier 3	
• data set	
• measures of center	
• measure of variability	

Standards Connections
6.SP.3 → 6.SP.5

6.SP.3 Examples:

Example: Representing Data and Finding Measures of Center.

Consider the data shown in the following dot plot of the scores for organization skills for a group of students.

a. How many students are represented in the data set?
b. What are the mean and median of the data set? Compare the mean and median.
c. What is the range of the data? What does this value tell you?

Solution:

a. Since there are 19 data points (represented by X's) in the set, there are 19 students represented.
b. The mean of the data set can be found by adding all of the data values (scores) and dividing by 19 (the calculation below is recorded as [score] × [number of students with that score])

$$\frac{0(1)+1(1)+2(2)+3(6)+4(4)+5(3)+6(2)}{19}=\frac{66}{19}\approx 3.47$$

The median of the data set appears to be 3. To check this, we can line up the data values and look for the center:

0, 1, 2, 2, 3, 3, 3, 3, 3, **3**, 4, 4, 4, 4, 5, 5, 5, 6, 6

The median is indeed 3, since there are 9 data values to the left and 9 values to the right of 3. The mean is greater than the median, which makes sense because the data is slightly skewed to the right.
c. The range of the data is 6, which happens to coincide with the range of scores possible.

Describing Data.

The *measure of center* gives a numerical value to represent the center of the data (e.g., midpoint of an ordered list or the balancing point). The *range* provides a single number that describes how the values vary across the data set. Another characteristic of a data set is the measure of *variability* (or spread) of the values.

6th Grade – CCSS for Mathematics

6.SP.A Develop understanding of statistical variability.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Essential Skills and Concepts:
☐ Understand and explain measures of center
☐ Understand and explain measures of variation

Question Stems and Prompts:
✓ What is the difference between measures of center and a measure of variation?
✓ Describe a data set in terms of measures of center.
✓ Describe a data set in terms of measures of variability.

Vocabulary / Spanish Cognates
Tier 2
• value — valor
Tier 3
• data set
• measures of center
• measure of variability

Standards Connections
6.SP.3 → 6.SP.5

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Describing Data.

The measure of center gives a numerical value to represent the center of the data (e.g., midpoint of an ordered list or the balancing point). The range provides a single number that describes how the values vary across the data set. Another characteristic of a data set is the measure of variability (or spread) of the values.

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6th Grade – CCSS for Mathematics

6.SP.A.3

Standard Explanation

Measures of Center

Given a set of data values, students summarize the measure of center with the median or mean (6.SP.3). The median is the value in the middle of an ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it. When there is an even number of data values, the median is the arithmetic average of the two in the middle.

The *mean* is the arithmetic average: the sum of the values in a data set divided by the number of data values in the set. The mean measures center in the sense that it is the hypothetical value that each data point would equal if the total of the data values were redistributed equally. Students can develop an understanding of what the mean represents by redistributing data sets to be level or fair (creating an equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (reflecting the idea of a balance point).

Measures of Variability

In grade six, variability is measured using the interquartile range or the mean absolute deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. In a box plot, it represents the length of the box and is not affected by outliers. Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference, or from reading a box plot. Mean absolute deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean, and then finding the average of these deviations. Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6.SP.A.3

Standard Explanation

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6.SP.B Summarize and describe distributions.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Essential Skills and Concepts:
- ☐ Summarize numerical data
- ☐ Display data in plots on a number line
- ☐ Display data in dot plots
- ☐ Display data in histograms
- ☐ Display data in box plots

Question Stems and Prompts:
- ✓ What does the data set represent?
- ✓ How can the distributions be described?

Vocabulary	**Spanish Cognates**
Tier 3	
• number line	línea de número
• dot plots	
• histogram	histograma
• box plots	

Standards Connections
6.SP.4 – 6.SP.5

6.SP.4 Examples:
Students can use applets to create data displays. For example:
- Box Plot Tool
  http://illuminations.nctm.org/ActivityDetail.aspx?ID=77
- Histogram Tool
  http://illuminations.nctm.org/ActivityDetail.aspx?ID=78
  (NCTM Illuminations 2013)

Graphical Displays of Data in Grade Six.
- *Dot plots* are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.
- A *histogram* shows the distribution of data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique.
- A *box plot* shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (*minimum, lower quartile, median, upper quartile,* and *maximum*). These values give a summary of the shape of a distribution. Box plots display the degree of spread of the data and the skewness of the data and can help students compare two sets of data.

6th Grade – CCSS for Mathematics

6.SP.B Summarize and describe distributions.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Essential Skills and Concepts:
- ☐ Summarize numerical data
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- ☐ Display data in dot plots
- ☐ Display data in histograms
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6.SP.B Summarize and describe distributions.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Standard Explanation

Grade six students display data graphically using number lines, as well as dot plots, histograms, and box plot graphs (6.SP.4). Students learn to determine the appropriate graph to use to display data and how to read data from graphs generated by others. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

6.SP.4, 5c Illustrative Task:

- Puzzle Times, https://www.illustrativemathematics.org/illustrations/877

Each of the 20 students in Mr. Anderson's class timed how long it took them to solve a puzzle. Their times (in minutes) are listed below:

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Time (minutes)	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

a. Display the data using a dot plot.

b. Find the mean and median of the data. Does it surprise you that the values of the mean and median are not equal? Explain why or why not.

6th Grade – CCSS for Mathematics

6.SP.B Summarize and describe distributions.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Standard Explanation

Grade six students display data graphically using number lines, as well as dot plots, histograms, and box plot graphs (6.SP.4). Students learn to determine the appropriate graph to use to display data and how to read data from graphs generated by others. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

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Time (minutes)	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

a. Display the data using a dot plot.

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6th Grade – CCSS for Mathematics

6.SP.B Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Essential Skills and Concepts:

- ☐ Understand numerical data in terms of context
- ☐ Report number of observations
- ☐ Describe how an attribute is measured
- ☐ Describe deviations from the pattern

Question Stems and Prompts:

- ✓ How can the data collected be described?
- ✓ Describe the pattern of the data
- ✓ Describe how the data is measured

Vocabulary	**Spanish Cognates**
Tier 2	
• median	mediana
• mean	
Tier 3	
• interquartile range	rango intercuartil
• mean absolute deviation	
• variability	variabilidad
• data	datos

Standards Connections

6.SP.5 – 6.SP.4

6.SP.5 Examples:

Examples: Interpreting Data Displays.

1. Grade six students were collecting data for a math class project. They decided they would survey the other two grade six classes to determine how many video games each student owns. A total of 38 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution: Students might make a histogram with 4 ranges (0-9,10-19, 20-29, 30-29) to display the data. It appears from the histogram that the mean and median are somewhere between 10-19, since the data of so many students lies in this range. Relatively few students own more than 30 video games, in fact, further analysis may prove the data point 39 to be an outlier.

6.SP.B Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
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Standards Connections

6.SP.5 – 6.SP.4

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Solution: Students might make a histogram with 4 ranges (0-9,10-19, 20-29, 30-29) to display the data. It appears from the histogram that the mean and median are somewhere between 10-19, since the data of so many students lies in this range. Relatively few students own more than 30 video games, in fact, further analysis may prove the data point 39 to be an outlier.

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6th Grade – CCSS for Mathematics

6.SP.B.5

Standard Explanation

Students in grade six interpret data displays and determine measures of center and variability from them. They summarize numerical data sets in relation to their context. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

Example: Finding the IQR and MAD

In the example above, the data set was 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6. The median (3) separated the data set into an upper and lower 50%. By further separating these two subsets, we obtain the four *quartiles* (i.e., 25%-sized parts of the data set).

0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6

In this case, the IQR is 5 – 3 = 2, indicating that the middle 50% of values differ by no more than 2 units. This is reflected in the dot plot, as most of the data appears to be clustered around 3 and 4.

To find the MAD of the data set above, we'll round the mean to 3.5 to simply our calculations, and find that there are 6 possible deviations from the mean:

$$|0-3.5|, |1-3.5|, |2-3.5|, |3-3.5|, |4-3.5|, |5-3.5|, |6-3.5|,$$

resulting in the set of deviations 3.5, 2.5, .5, .5, 1.5, 2.5. When we find the average of these deviations, we obtain:

$$\frac{1(3.5)+1(2.5)+2(1.5)+6(.5)+4(.5)+3(1.5)+2(2.5)}{19} \approx 1.24.$$

This is interpreted as saying that on average, a student's score was 1.24 points away from the approximate mean of 3.5.

6.SP.4, 5c Illustrative Task:

- Puzzle Times, https://www.illustrativemathematics.org/illustrations/877

Each of the 20 students in Mr. Anderson's class timed how long it took them to solve a puzzle. Their times (in minutes) are listed below:

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Time (minutes)	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

a. Display the data using a dot plot.

b. Find the mean and median of the data. Does it surprise you that the values of the mean and median are not equal? Explain why or why not.

- Electoral College, https://www.illustrativemathematics.org/illustrations/1199

State	Electoral Votes	State	Electoral Votes	State	Electoral Votes
Alabama	9	Kentucky	8	North Dakota	3
Alaska	3	Louisiana	8	Ohio	18
Arizona	11	Maine	4	Oklahoma	7
Arkansas	6	Maryland	10	Oregon	7
California	55	Massachusetts	11	Pennsylvania	20
Colorado	9	Michigan	16	Rhode Island	4
Connecticut	7	Minnesota	10	South Carolina	9
Delaware	3	Mississippi	6	South Dakota	3
District of Columbia	3	Missouri	10	Tennessee	11
Florida	29	Montana	3	Texas	38
Georgia	16	Nebraska	5	Utah	6
Hawaii	4	Nevada	6	Vermont	3
Idaho	4	New Hampshire	4	Virginia	13
Illinois	20	New Jersey	14	Washington	12
Indiana	11	New Mexico	5	West Virginia	5
Iowa	6	New York	29	Wisconsin	10
Kansas	6	North Carolina	15	Wyoming	3

a. Which state has the most electoral votes? How many votes does it have?

b. Based on the given information, which state has the second highest population of qualified citizens?

c. Here is a dotplot of the distribution.

6.SP.B.5

Standard Explanation

Students in grade six interpret data displays and determine measures of center and variability from them. They summarize numerical data sets in relation to their context. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

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0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6

In this case, the IQR is 5 – 3 = 2, indicating that the middle 50% of values differ by no more than 2 units. This is reflected in the dot plot, as most of the data appears to be clustered around 3 and 4.

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Alaska	3	Louisiana	8	Ohio	18
Arizona	11	Maine	4	Oklahoma	7
Arkansas	6	Maryland	10	Oregon	7
California	55	Massachusetts	11	Pennsylvania	20
Colorado	9	Michigan	16	Rhode Island	4
Connecticut	7	Minnesota	10	South Carolina	9
Delaware	3	Mississippi	6	South Dakota	3
District of Columbia	3	Missouri	10	Tennessee	11
Florida	29	Montana	3	Texas	38
Georgia	16	Nebraska	5	Utah	6
Hawaii	4	Nevada	6	Vermont	3
Idaho	4	New Hampshire	4	Virginia	13
Illinois	20	New Jersey	14	Washington	12
Indiana	11	New Mexico	5	West Virginia	5
Iowa	6	New York	29	Wisconsin	10
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a. Which state has the most electoral votes? How many votes does it have?

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c. Here is a dotplot of the distribution.

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Resources for the CCSS 6^{th} Grade Bookmarks

California *Mathematics Framework,* adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

Common Core Standards Writing Team. Progressions for the Common Core State Standards in Mathematics Tucson, AZ: Institute for Mathematics and Education, University of Arizona (Drafts)

- K, Counting and Cardinality; K – 5 Operations and Algebraic Thinking (2011, May 29)
- K – 5, Number and Operations in Base Ten (2012, April 21)
- K – 3, Categorical Data; Grades 2 – 5, Measurement Data* (2011, June 20)
- K – 5, Geometric Measurement (2012, June 23)
- K – 6, Geometry (2012, June 23)
- Number and Operations – Fractions, 3 – 5 (2013, September 19)

Illustrative Mathematics™ was originally developed at the University of Arizona (2011), nonprofit corporation (2013), Illustrative Tasks, http://www.illustrativemathematics.org/

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

North Carolina Department of Public Instruction, Instructional Support Tools for Achieving New Standards, Math Unpacking Standards 2012, http://www.ncpublicschools.org/acre/standards/common-core-tools/ - unmath

Common Core Flipbooks 2012, Kansas Association of Teachers of Mathematics (KATM) http://www.katm.org/baker/pages/common-core-resources.php

6^{th} Grade – CCSS for Mathematics

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North Carolina Department of Public Instruction, Instructional Support Tools for Achieving New Standards, Math Unpacking Standards 2012, http://www.ncpublicschools.org/acre/standards/common-core-tools/ - unmath

Common Core Flipbooks 2012, Kansas Association of Teachers of Mathematics (KATM) http://www.katm.org/baker/pages/common-core-resources.php

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Arizona's College and Career Ready Standards – Mathematics – Kindergarten, Arizona Department of Education – High Academic Standards for Students Arizona's College and Career Ready Standards – Mathematics, State Board Approved June 2010 October 2013 Publication, http://www.azed.gov/azccrs/mathstandards/

Howard County Public School System, Elementary Mathematics Office, Standards for Mathematical Practice for Parents, Draft 2011, https://grade3commoncoremath.wikispaces.hcpss.org/file/view/SFMP for Parents.docx/286906254/SFMP for Parents.docx

Howard County Public School System, Elementary and Secondary Mathematics Offices, Wiki Content and Resources, Elementary by grade level https://grade5commoncoremath.wikispaces.hcpss.org/home, and Secondary https://secondarymathcommoncore.wikispaces.hcpss.org

Long Beach Unified School District, Math Cognates, retrieved on 7/14/14, http://www.lbschools.net/Main_Offices/Curriculum/Areas/Mathematics/XCD/ListOfMathCognates.pdf

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