Supporting California's Standards

Mathematics Bookmarks

Standards Reference to Support Planning and Instruction

7th Grade

Tulare County Office of Education

Tim A. Hire, County Superintendent of Schools

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Grade-Level Introduction

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

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(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

FLUENCY
In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., “fluently” multiply multi-digit whole numbers using the standard algorithm (5.NBT.5▲). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary. The word “fluent” is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.

California *Mathematics Framework,* adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

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Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical Practices

1. **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".

Students:	Teachers:
• Analyze and explain the meaning of the problem • Actively engage in problem solving (Develop, carry out, and refine a plan) • Show patience and positive attitudes • Ask if their answers make sense • Check their answers with a different method	• Pose rich problems and/or ask open ended questions • Provide wait-time for processing/finding solutions • Circulate to pose probing questions and monitor student progress • Provide opportunities and time for cooperative problem solving and reciprocal teaching

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2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

Students:	Teachers:
• Represent a problem with symbols • Explain their thinking • Use numbers flexibly by applying properties of operations and place value • Examine the reasonableness of their answers/calculations	• Ask students to explain their thinking regardless of accuracy • Highlight flexible use of numbers • Facilitate discussion through guided questions and representations • Accept varied solutions/representations

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3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).

In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?". They explain their thinking to others and respond to others' thinking.

Students:	Teachers:
• Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying and probing questions	• Provide opportunities for students to listen to or read the conclusions and arguments of others • Establish and facilitate a safe environment for discussion • Ask clarifying and probing questions • Avoid giving too much assistance (e.g., providing answers or procedures)

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4. **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

Students:	Teachers:
• Make reasonable guesses to explore their ideas • Justify solutions and approaches • Listen to the reasoning of others, compare arguments, and decide if the arguments of others makes sense • Ask clarifying questions	• Allow time for the process to take place (model, make graphs, etc.) • Model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) • Make appropriate tools available • Create an emotionally safe environment where risk taking is valued • Provide meaningful, real world, authentic, performance-based tasks (non traditional work problems)

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5. **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.

Students:	Teachers:
• Select and use tools strategically (and flexibly) to visualize, explore, and compare information • Use technological tools and resources to solve problems and deepen understanding	• Make appropriate tools available for learning (calculators, concrete models, digital resources, pencil/paper, compass, protractor, etc.) • Use tools with their instruction

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6. **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.

Students:	Teachers:
• Calculate accurately and efficiently • Explain their thinking using mathematics vocabulary • Use appropriate symbols and specify units of measure	• Recognize and model efficient strategies for computation • Use (and challenging students to use) mathematics vocabulary precisely and consistently

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7. **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 2(3 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.

Students:	Teachers:
• Look for, develop, and generalize relationships and patterns • Apply reasonable thoughts about patterns and properties to new situations	• Provide time for applying and discussing properties • Ask questions about the application of patterns • Highlight different approaches for solving problems

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8. **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

Students:	Teachers:
• Look for methods and shortcuts in patterns and repeated calculations • Evaluate the reasonableness of results and solutions	• Provide tasks and problems with patterns • Ask about possible answers before, and reasonableness after computations

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Grade 7 Overview

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations
- Investigate chance processes and develop, use, and evaluate probability models.

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major3 [m] clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. The ▲ symbol will indicate standards in a Major Cluster in the narrative.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade Supporting [s] clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

California *Mathematics Framework,* adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

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7th Grade – CCSS for Mathematics

CCSS Where to Focus Grade 7 Mathematics

Not all of the content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have a greater emphasis is not to say that anything in the standards can be safely neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 7

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters ☐ Supporting Clusters ○ Additional Clusters

7.RP.A	■	Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.NS.A	■	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
7.EE.A	■	Use properties of operations to generate equivalent expressions.
7.EE.B	■	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
7.G.A	○	Draw, construct and describe geometrical figures and describe the relationships between them.
7.G.B	○	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
7.SP.A	☐	Use random sampling to draw inferences about a population.
7.SP.B	○	Draw informal comparative inferences about two populations.
7.SP.C	☐	Investigate chance processes and develop, use, and evaluate probability models.

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

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Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

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7th Grade – CCSS for Mathematics

7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.*

Essential Skills and Concepts:
- ☐ Compute unit rates with ratios of fractions
- ☐ Compute rates with like and different units
- ☐ Compute ratios involving lengths, areas, and other quantities

Question Stems and Prompts:
- ✓ What is the unit rate of the ratio?
- ✓ What is the difference between a unit rate and ratio? How are they similar?
- ✓ What is the relationship between the two units?
- ✓ What does the ratio represent?

Vocabulary	**Spanish Cognates**
Tier 3	
• unit rate	
• fractions	fracciones
• length	longitud
• area	área
• tape diagrams	
• double number lines	
• ratio tables	

Standards Connections
6.RP.2 → *7.RP.1, 7.RP.1* → 7.RP.2

7.RP.1 Illustrative Tasks:
- Track Practice, https://www.illustrativemathematics.org/illustrations/82

Angel and Jayden were at track practice. The track is $\frac{2}{5}$ kilometers around.

- Angel ran 1 lap in 2 minutes.
- Jayden ran 3 laps in 5 minutes.

a. How many minutes does it take Angel to run one kilometer? What about Jayden?
b. How far does Angel run in one minute? What about Jayden?
c. Who is running faster? Explain your reasoning.

- Molly's Run, https://www.illustrativemathematics.org/illustrations/828

Molly runs $\frac{1}{3}$ of a mile in 4 minutes.

a. If Molly continues at the same speed, how long will it take her to run one mile?
b. Draw and label a picture showing why your answer to part (a) makes sense.

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- ✓ What is the difference between a unit rate and ratio? How are they similar?
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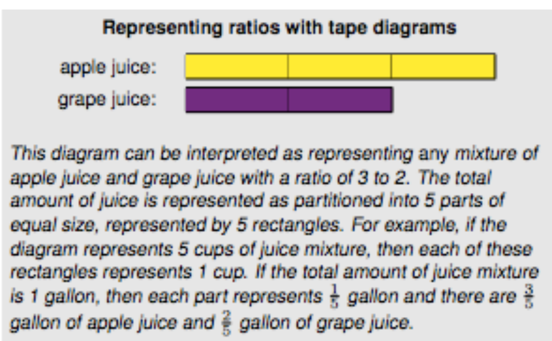
7^{th} Grade – CCSS for Mathematics

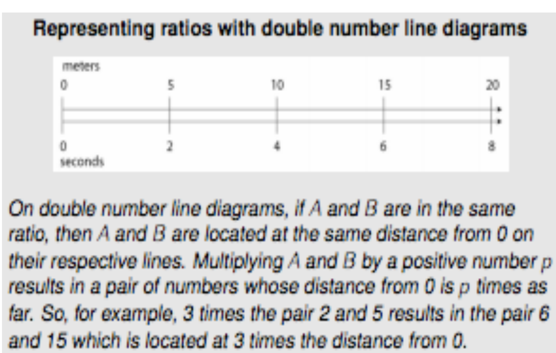
7.RP.A.1

Standard Explanation

A critical area of instruction in grade seven is developing an understanding and application of proportional relationships, including percentages. In grade seven, students extend their reasoning about ratios and proportional relationships in several ways. Students use ratios in cases that involve pairs of rational number entries, and they compute associated rates. They identify unit rates in representations of proportional relationships. They work with equations in two variables to represent and analyze proportional relationships. They also solve multi-step ratio and percent problems, such as problems involving percent increase and decrease.

Important in grade seven is the concept of the unit rate associated with a ratio. For a ratio a: b with a, $b \neq 0$, the unit rate is the number a/b. In sixth grade, students worked primarily with ratios involving whole number quantities. In addition, they discovered what it meant to have equivalent ratios. In grade seven, students will find unit rates in ratios involving fractional quantities (7.RP.1▲). For example, when a recipe calls for 1 ½ cups of sugar and 3 cups of flour, this results in a unit rate of $\frac{1\frac{1}{2}}{3} = \frac{3}{6}$. The fact that any pair of quantities in a proportional relationship can be divided to find the unit rate will be useful when solving problems involving proportional relationships, as this will allow students to set up an equation with equivalent fractions and use reasoning about equivalent fractions to solve them.





(The University of Arizona Progressions Documents for the Common Core Math Standards [Progressions] 6-7 Ratios and Proportional Relationships [RP] 2011).

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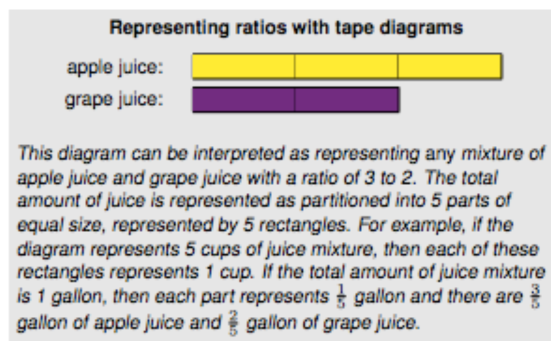
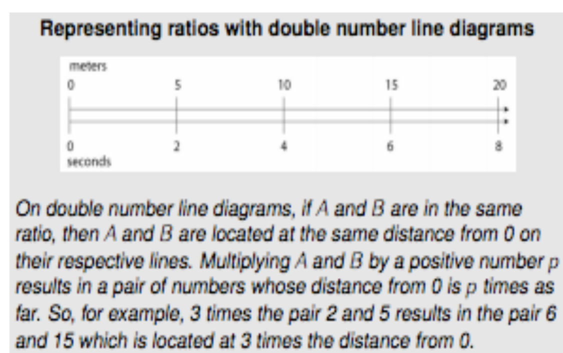

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7th Grade – CCSS for Mathematics

7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.

Essential Skills and Concepts:

- ☐ Understand and identify when two quantities are proportional by testing for equivalent ratios
- ☐ Use graphs, equations, and diagrams to identify the constant of proportionality
- ☐ Write equations representing proportional relationships
- ☐ Explain what points on the graph of a proportional relationship mean, including the origin and unit rate

Question Stems and Prompts:

- ✓ What is a proportional relationship?
- ✓ Do the two quantities form a proportional relationship? How do you know?
- ✓ What is the constant of proportionality?
- ✓ Describe this graph of a proportional relationship. What do different points on the graph represent?

Vocabulary	**Spanish Cognates**
Tier 3	
• equivalent ratios	relación de equivalentes
• proportion	proporción
• proportional relationship	relación proporcional
• unit rate	
• constant of proportionality	constante de proporcionalidad
• ratio table	
• coordinate plane	plano de coordenadas

Standards Connections

6.RP.2, 6.RP.3, 7.RP.1 → *7.RP.2*
7.RP.2 – 7.EE.4a
7.RP.2 → 7.RP.3, 7.G.1

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Standards Connections

6.RP.2, 6.RP.3, 7.RP.1 → *7.RP.2*
7.RP.2 – 7.EE.4a
7.RP.2 → 7.RP.3, 7.G.1

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7.RP.A.2

Standard Explanation

In grade six, students worked with many examples of proportional relationships and represented them numerically, pictorially, graphically, and with equations in simple cases. In grade seven, students continue this work, but they examine more closely the characteristics of proportional relationships. In particular, they begin to identify:

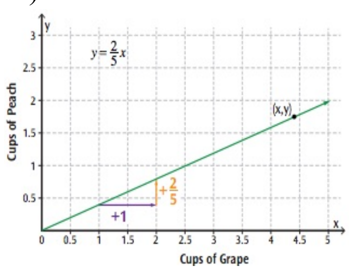
- When proportional quantities are represented in a table, pairs of entries represent equivalent ratios.
- The graph of a proportional relationship lies on a straight line that passes through the point (0,0) (indicating that when one quantity is 0, so is the other).[5]
- Equations of proportional relationships in a ratio of $a : b$ always take the form $y = k \cdot x$, where k is the constant $\frac{b}{a}$ if the variables x and y are defined so that the ratio $x : y$ is equivalent to $a : b$. (The number k is also known as the constant of proportionality). (7.RP.2▲).

Thus a first, and often overlooked, step is for students to decide when and why two quantities are actually in a proportional relationship (7.RP.2a▲). They can do this by checking the characteristics listed above, or by using reasoning (e.g., a runner's heart rate might increase steadily as he runs faster, but his heart rate when he is standing still is not 0 beats per minute, hence running speed and heart rate are not proportional). (Adapted from Progressions 6-7 RP 2011).

The formal reasoning behind this principle and the next one is not expected until grade eight (see 8.EE.B). But students will notice and informally use both principles in grade seven.

Students use a variety of methods to solve problems involving proportional relationships. They should have opportunities to solve these problems with strategies such as making tape diagrams and double number lines, using tables, using rates, and by relating proportional relationships to equivalent fractions as described above (CA Mathematics Framework, Adopted Nov. 6, 2013).

	x Cups of Grape	y Cups of Peach
Batch A	0	0
Batch B	5	2
Batch C	1	$\frac{2}{5}$
Batch D	2	$2 \cdot \frac{2}{5}$
Batch E	3	$3 \cdot \frac{2}{5}$
Batch F	4	$4 \cdot \frac{2}{5}$
Any batch made according to the recipe	x	$x \cdot \frac{2}{5}$



Deriving an Equation. Both the table and the graph show that for every 1 cup of grape juice added, $\frac{2}{5}$ cup of peach juice is added. Thus, starting with an empty bowl, when x cups of grape juice are added, $\frac{2}{5}x$ cup of peach juice must be added. On the graph, this corresponds to the fact that, when starting from (0,0), every movement horizontally of x units results in a vertical movement of $\frac{2}{5}x$ units. In either case, the equation becomes $y = \frac{2}{5}x$.

Adapted from UA Progressions Documents 2011c.

7th Grade – CCSS for Mathematics

7.RP.A.2

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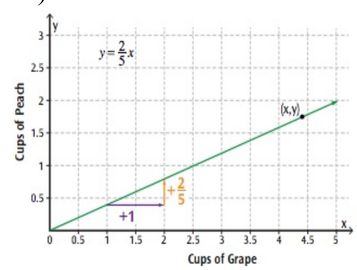

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7th Grade – CCSS for Mathematics

7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error***.**

Essential Skills and Concepts:
- ☐ Solve real-world and mathematical ratio problems
- ☐ Solve real-world and mathematical percent problems
- ☐ Use proportional relationships to solve problems

Question Stems and Prompts:
- ✓ What information do know about this problem and what are you asked to find?
- ✓ How are solving tax problems, gratuities, and commission problems similar? How are they different?

Vocabulary	**Spanish Cognates**
Tier 3	
• simple interest	interés simple
• tax	
• markups	
• markdowns	
• gratuities	
• commissions	comisiónes
• fee	
• percent	por ciento
• tape diagrams	
• double number lines	

Standards Connections
7.RP.2 → *7.RP.3, 7.RP.3* → 7.SP.6, 7.SP.7, 7.SP.8

7.RP.3 Examples:
Gas prices are projected to increase by 124% by April 2015. A gallon of gas currently costs \$3.80. What is the projected cost of a gallon of gas for April 2015?
Solution: "The original cost of a gallon of gas is \$3.80. An increase of 100% means that the cost will double. Another 24% will need to be added to figure out the final projected cost of a gallon of gas. Since 25% of \$3.80 is about \$0.95, the projected cost of a gallon of gas should be around \$8.15."

$$\$3.80 + 3.80 + (0.24 \bullet 3.80) = 2.24 \times 3.80 = \$8.15$$

100%	100%	24%
\$3.80	\$3.80	?

A sweater is marked down 33% off the original price. The original price was \$37.50. What is the sale price of the sweater before sales tax?
Solution: The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

37.50 Original Price of Sweater	
33% of 37.50 Discount	67% of 37.50 Sale Price of Sweater

Adapted from ADE and NCDPI.

7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error***.**

Essential Skills and Concepts:
- ☐ Solve real-world and mathematical ratio problems
- ☐ Solve real-world and mathematical percent problems
- ☐ Use proportional relationships to solve problems

Question Stems and Prompts:
- ✓ What information do know about this problem and what are you asked to find?
- ✓ How are solving tax problems, gratuities, and commission problems similar? How are they different?

Vocabulary	**Spanish Cognates**
Tier 3	
• simple interest	interés simple
• tax	
• markups	
• markdowns	
• gratuities	
• commissions	comisiónes
• fee	
• percent	por ciento
• tape diagrams	
• double number lines	

Standards Connections
7.RP.2 → *7.RP.3, 7.RP.3* → 7.SP.6, 7.SP.7, 7.SP.8

7.RP.3 Examples:
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7th Grade – CCSS for Mathematics

7.RP.A.3

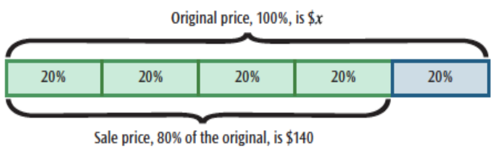
Standard Explanation

In grade six, students used ratio tables and unit rates to solve percent problems. In grade seven, students extend their work to solve multi-step ratio and percent problems (7.RP.3▲). They explain or show their work using a representation (e.g., numbers, words, pictures, physical objects, or equations) and verify that their answers are reasonable. Models help students identify parts of the problem and how values are related (MP. 1, MP.3 and MP.4). For percentage increase and decrease, students identify the original value, determine the difference, and compare the difference in the two values to the starting value.

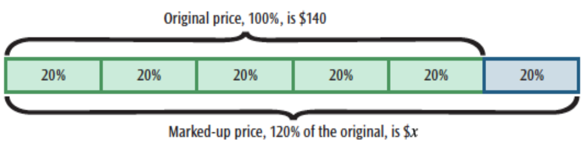
Problems involving percentage increase or percentage decrease require careful attention to the referent whole. For example, consider the difference in these two problems:

Skateboard Problem 1. The problem can be represented with a tape diagram. Students reason that since 80% is \$140, 20% is $\$140 \div 4 = \$35$, so 100% is then $5 \times \$35 = \$175$.

Equivalently, $0.80 \cdot x = 140$, so that $x = 140 \div 0.08$, or $x = 140 \div \frac{4}{5} = 140 \times \frac{5}{4} = 35 \times 5 = 175$.



Skateboard Problem 2. This problem can be represented with a tape diagram as well. Students can reason that since 100% is \$140, 20% is $\$140 \div 5 = 28$, so 120% is then $6 \times \$28 = \$168$. Equivalently, $x = (1.20)(140)$, so $x = 168$.



Focus, Coherence, and Rigor

Problems involving proportional relationships support mathematical practices as students make sense of problems (**MP.1**), reason abstractly and quantitatively (**MP.2**), and model proportional relationships (**MP.4**). For example, for modeling purposes, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building.

Adapted from PARCC 2012.

(CA Mathematics Framework, Adopted Nov. 6, 2013)

7.RP.3 Illustrative Tasks:

- Finding a 10% Increase, https://www.illustrativemathematics.org/illustrations/132

5,000 people visited a book fair in the first week. The number of visitors increased by 10% in the second week. How many people visited the book fair in the second week?

- Two-School Dance, https://www.illustrativemathematics.org/illustrations/886

There are 270 students at Colfax Middle School, where the ratio of boys to girls is 5:4. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is 4:5. The two schools hold a dance and all students from both schools attend. What fraction of the students at the dance are girls?

7.RP.A.3

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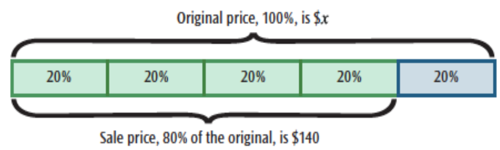

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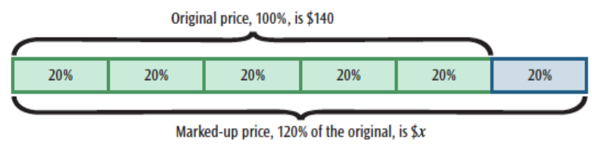

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7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.

Essential Skills and Concepts:
- ☐ Describe situations where opposites are added to make zero, understanding that the sum of a number and its opposite are zero (additive inverse)
- ☐ Understand positive and negative directions
- ☐ Add and subtract rational numbers
- ☐ Understand subtraction of rational numbers as adding the additive inverse

Question Stems and Prompts:
- ✓ Describe a situation that can be represented by the problem.
- ✓ What are the positive and negative directions?
- ✓ Describe how to add/subtract rational numbers.
- ✓ What is the additive inverse? How do you use it when adding and subtracting rational numbers?

Vocabulary	**Spanish Cognates**
Tier 3	
• rational numbers	números racionales
• opposites	
• positive direction	dirección positiva
• negative direction	dirección negativa
• absolute value	valor absoluto
• additive inverse	inverso aditivo

Standards Connections
5.NF.1, 6.NS.5, 6.NS.6a, 6.NS.7c → *7.NS.1*
7.NS.1 → 7.NS.2, 7.NS.3

7th Grade – CCSS for Mathematics

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7.NS.1 → 7.NS.2, 7.NS.3

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7.NS.A.1

Standard Explanation

Seventh grade students extend addition, subtraction, multiplication, and division to all rational numbers by applying these operations to both positive and negative numbers. Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade eight, expanding to become the real numbers by the introduction of irrational numbers. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in grade seven depends on rational number arithmetic, fluency with rational number arithmetic should be the goal in grade seven (Adapted from PARCC 2012).

Previously in grade six, students learned that the absolute value of a rational number is its distance from zero on the number line. In grade seven, students represent addition and subtraction with positive and negative rational numbers on a horizontal or vertical number line diagram (7.NS.1 a-c▲). Students add and subtract, understanding $p + q$ as the number located a distance $|q|$ from p on a number line, in the positive or negative direction, depending on whether q is positive or negative. They demonstrate that a number and its opposite have a sum of 0 (i.e. they are additive inverses), and they understand subtraction of rational numbers as adding the additive inverse. (MP.2, MP.4, and MP.7) Students' work with signed numbers began in grade six, where they experienced situations in which positive and negative numbers represented (for example) credits or debits to an account, positive or negative charges, or increases or decreases, all relative to a 0. Now, students realize that in each of these situations, a positive quantity and negative quantity of the same absolute value add to make 0 (7.NS.1 a▲).

For instance, the positive charge of 5 protons would neutralize the negative charge of 5 electrons, and we represent this as: $(+5) + (-5) = 0$. Students recognize that +5 and −5 are "opposites" as described in grade six, located the same distance from 0 on a number line. But they reason further here that opposites have the relationship that a number a and its opposite $-a$ always combine to make 0: $a + (-a) = 0$. This crucial fact lays the foundation for understanding addition and subtraction of signed numbers.

Students' work with rational numbers should include rational numbers in different forms—positive and negative fractions, decimals, and whole numbers (including combinations). Integers might be used to introduce the ideas of signed number operations, but student work and practice should not be limited to integer operations. If students learn to compute $\mathbf{4 + (-8)}$, but not $\mathbf{4 + (-\frac{1}{3})}$ then they are not learning the rational number system. (CA *Mathematics Framework*, adopted Nov. 6, 2013)

7th Grade – CCSS for Mathematics

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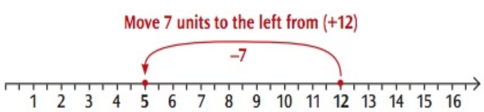
7.NS.A.1 Continued

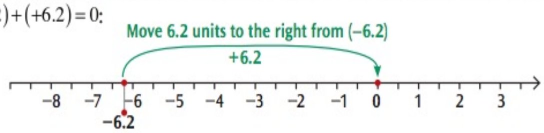
Addition of Rational Numbers

Through experiences starting with whole numbers and their opposites (i.e., starting with integers only), students can develop the understanding that like quantities can be combined. That is, two positive quantities combine to become a “more positive” quantity, as in (+5) + (+7) = +12 , and two negative quantities combine to become a “more negative” quantity, as in (– 2) + (– 10) = – 12. When addition problems have mixed signs, students see that positive and negative quantities combine as necessary to partially make zeros (i.e., they “cancel” each other), and the appropriate amount of positive or negative charge remains.

Eventually, students come to realize that when adding two numbers with different signs, the sum is equal to the difference of the absolute values of the two numbers and has the same sign as the number with the larger absolute value. This understanding eventually replaces the kinds of calculations shown above, which are meant to illustrate concepts rather than serving as practical computation methods. When students use a number line to represent the addition of integers, they can develop a general understanding that the sum $p + q$ is the number found when moving a total of $|q|$ units from p to the right if q is positive, and to the left if q is negative (7.NS.1b). The number line below represents (+12) + (– 7) :



The concept is particularly transparent for quantities that combine to become 0, as illustrated in the example $(-6.2)+(+6.2)=0$:



Subtraction of Rational Numbers

When subtracting rational numbers, the most important concept for students to grasp is that gives the same result as $p +$ $(– q)$; that is, subtracting q is equivalent to adding the opposite of q. Students have most likely already noticed that with sums such as 10 + (– 2), the result was the same as finding the difference, 10 – 2. For subtraction of quantities with the same sign, teachers may find it helpful to employ typical understandings of subtraction as “taking away” or comparing to an equivalent addition problem, as in (– 12) – (– 7), meaning to “take away –7 from –12,” and compare this with (– 12) + 7. With an understanding that these numbers represent negative charges, the answer of –5 is arrived at fairly easily. However, by comparing this subtraction expression with the addition expression (– 12) + 7, students see that both result in –5. Through many examples, students can generalize these results to understand that [7.NS.1c▲].

7th Grade – CCSS for Mathematics

7.NS.A.1 Continued

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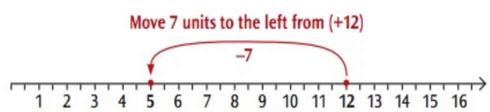

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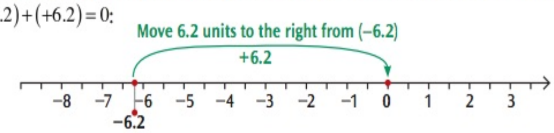

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7.NS.A.1 Continued

Common Concrete Models for Addition and Subtraction of Rational Numbers

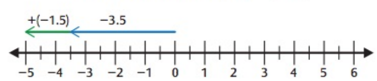
Several different concrete models may be used to represent rational numbers and operations with rational numbers. It is important for teachers to understand that all such concrete models have advantages and disadvantages, and therefore care should be taken when introducing these models to students. Not every model will lend itself well to representing every aspect of operations with rational numbers. Brief descriptions of some common concrete models are provided below.

Common Concrete Models for Representing Signed Rational Numbers 7.NS.1d▲ (MP.5)

1. **Number Line Models (Vector Models).** A number line is used to represent the set of all rational numbers, and directed line segments (i.e., *vectors,* which look like arrows) are used to represent numbers. The length of the arrow is the absolute value of the number, and the direction of the arrow tells the sign of the number. Thus, the arrow emanating from 0 to –3.5 on the number line represents the number –3.5.

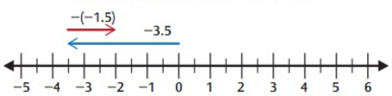
   Addition is then represented by placing arrows head to tail and looking at the number to which the final arrow points.

   Number Line Model for $(-3.5)+(-1.5)=-5$



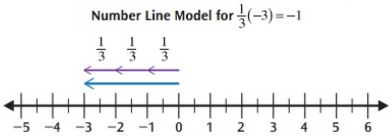
   Subtraction is equivalent to adding the opposite, so we can represent $a-b$ by reversing the arrow for b and then adding it to a.

   Number Line Model for $(-3.5)-(-1.5)=-2$



   Multiplication is interpreted as scaling. For example, the product $\frac{1}{3}(-3)$ can be interpreted as a vector one-third the length of the vector (-3) in the same direction. That is, $\frac{1}{3}(-3)=-1$.

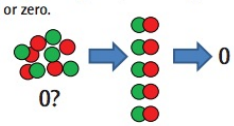
   Number Line Model for $\frac{1}{3}(-3)=-1$



Grade seven marks the culmination of the arithmetic learning progression for rational numbers. By the end of seventh grade, students' arithmetic repertoire includes adding, subtracting, multiplying, and dividing with rational numbers including whole includes numbers, fractions, decimals, and signed numbers.

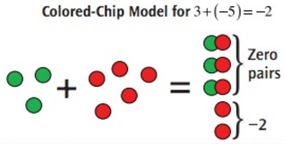
Common Concrete Models for Representing Signed Rational Numbers—7.NS.1d *(continued)*

2. **Colored-Chip Models.** Chips of one color are used to represent positive units, and chips of another color are used to represent negative units (note that plus and minus signs are sometimes written on the chips). These models make it easy to represent units that are combined, and they are especially illustrative when positive and negative units are combined to create "zero pairs" (sometimes referred to as *neutral pairs*), representing that $a+(-a)=0$. A disadvantage of these models is that multiplication and division are more difficult to represent, and chip models are typically used only to represent integer quantities (i.e., it is difficult to extend them to fractional quantities). Also, some imagination is required to view a pile of colored chips as representing "nothing" or zero.



   An equal number of positive and negative chips form zero pairs, representing zero.

   Colored-Chip Model for $3+(-5)=-2$



7^{th} Grade – CCSS for Mathematics

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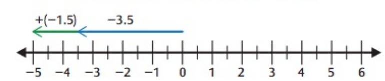
1. Number Line Models (Vector Models). A number line is used to represent the set of all rational numbers, and directed line segments (i.e., vectors, which look like arrows) are used to represent numbers. The length of the arrow is the absolute value of the number, and the direction of the arrow tells the sign of the number. Thus, the arrow emanating from 0 to –3.5 on the number line represents the number –3.5.

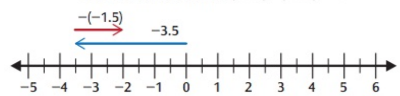
Addition is then represented by placing arrows head to tail and looking at the number to which the final arrow points.

Number Line Model for $(-3.5)+(-1.5)=-5$



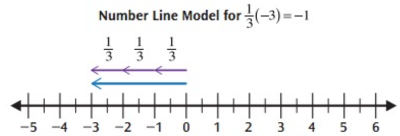
Subtraction is equivalent to adding the opposite, so we can represent $a-b$ by reversing the arrow for b and then adding it to a.

Number Line Model for $(-3.5)-(-1.5)=-2$



Multiplication is interpreted as scaling. For example, the product $\frac{1}{3}(-3)$ can be interpreted as a vector one-third the length of the vector (-3) in the same direction. That is, $\frac{1}{3}(-3)=-1$.

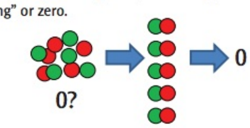
Number Line Model for $\frac{1}{3}(-3)=-1$



Grade seven marks the culmination of the arithmetic learning progression for rational numbers. By the end of seventh grade, students' arithmetic repertoire includes adding, subtracting, multiplying, and dividing with rational numbers including whole includes numbers, fractions, decimals, and signed numbers.

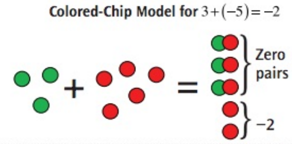
Common Concrete Models for Representing Signed Rational Numbers—7.NS.1d (continued)

2. Colored-Chip Models. Chips of one color are used to represent positive units, and chips of another color are used to represent negative units (note that plus and minus signs are sometimes written on the chips). These models make it easy to represent units that are combined, and they are especially illustrative when positive and negative units are combined to create "zero pairs" (sometimes referred to as neutral pairs), representing that $a+(-a)=0$. A disadvantage of these models is that multiplication and division are more difficult to represent, and chip models are typically used only to represent integer quantities (i.e., it is difficult to extend them to fractional quantities). Also, some imagination is required to view a pile of colored chips as representing "nothing" or zero.



An equal number of positive and negative chips form zero pairs, representing zero.

Colored-Chip Model for $3+(-5)=-2$



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7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.2
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real world contexts.
c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Essential Skills and Concepts:
- ☐ Multiply and divide rational numbers, including integers
- ☐ Describe real world contexts that involve the multiplication and division of rational numbers
- ☐ Convert rational numbers into decimals using long division
- ☐ Understand that rational numbers in decimal form terminate or repeat

Question Stems and Prompts:
- ✓ What do you know about the decimal forms of rational numbers?
- ✓ What properties of operations can be used to multiply and divide rational numbers?

Vocabulary	**Spanish Cognates**
Tier 2	
• terminate	terminar
• repeat	repetir
Tier 3	
• integers	
• rational numbers	numeros racionales
• distributive property	propiedad distributiva
• decimal form	forma decimal

Standards Connections
5.NF.3, 5.NF.4, 6.NS.1, 7.NS.1 → *7.NS.2*
7.NS.2 → 7.NS.3

7th Grade – CCSS for Mathematics

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7.NS.2
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

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Vocabulary	**Spanish Cognates**
Tier 2	
• terminate	terminar
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• integers	
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Standards Connections
5.NF.3, 5.NF.4, 6.NS.1, 7.NS.1 → *7.NS.2*
7.NS.2 → 7.NS.3

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7th Grade – CCSS for Mathematics

7.NS.A.2

Standard Explanation

Students continue to develop their understanding of operations with rational numbers by seeing that multiplication and division can be extended to signed rational numbers (7.NS.2▲). For instance, students can understand that in an account balance model, (−3)($40.00) can be thought of as a record of 3 groups of debits (indicated by the negative sign) of $40.00 each, resulting in a total contribution to the balance of −$120.00. In a vector model, students can interpret the expression (2.5)(−7.5) as the vector that points in the same direction as the vector representing −7.5, but is 2.5 times as long. Interpreting multiplication of two negatives in everyday terms can be troublesome, since negative money cannot be withdrawn from a bank. In a vector model, multiplying by a negative number reverses the direction of the vector (in addition to any stretching or compressing of the vector). Division is often difficult to interpret in everyday terms as well, but can always be understood mathematically in terms of multiplication, specifically as multiplying by the reciprocal.

Multiplication of Signed Rational Numbers

In general, multiplication of signed rational numbers is performed as with fractions and whole numbers, but according to the following rules for determining the sign of the product:

1. Different signs: $(-a) \times b = -ab$
2. Same signs: $(-a) \times (-b) = ab$

In these equations, both a and b can be positive, negative, or zero. Of particular importance is that $-1 \cdot a = -a$, that is, multiplying a number by negative one gives the opposite of the number. The first of these rules can be understood in terms of models as mentioned above. The second can be understood as being a result of properties of operations (refer to "A Derivation of the Fact that (−1)(−1) = 1" below). Students can also become comfortable with rule (ii) by examining patterns in products of signed numbers, such as in the table below, though this does not constitute a valid mathematical proof.

After arriving at a general understanding of these two rules for multiplying signed numbers, students can multiply any rational numbers by finding the product of the absolute values of the numbers and then determining the sign according to the rules.

Division of Rational Numbers

The relationship between multiplication and division allows students to infer the sign of the quotient of two rational numbers. Otherwise, division is performed as usual with whole numbers and fractions, with the sign to be determined (CA *Mathematics Framework*, adopted Nov. 6, 2013).

7.NS.A.2

Standard Explanation

Students continue to develop their understanding of operations with rational numbers by seeing that multiplication and division can be extended to signed rational numbers (7.NS.2▲). For instance, students can understand that in an account balance model, (−3)($40.00) can be thought of as a record of 3 groups of debits (indicated by the negative sign) of $40.00 each, resulting in a total contribution to the balance of −$120.00. In a vector model, students can interpret the expression (2.5)(−7.5) as the vector that points in the same direction as the vector representing −7.5, but is 2.5 times as long. Interpreting multiplication of two negatives in everyday terms can be troublesome, since negative money cannot be withdrawn from a bank. In a vector model, multiplying by a negative number reverses the direction of the vector (in addition to any stretching or compressing of the vector). Division is often difficult to interpret in everyday terms as well, but can always be understood mathematically in terms of multiplication, specifically as multiplying by the reciprocal.

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7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

Essential Skills and Concepts:

- ☐ Solve real world problems involving rational numbers using the four operations
- ☐ Solve mathematical problems involving rational numbers using the four operations

Question Stems and Prompts:

- ✓ Write an equation to represent the real world problem.
- ✓ Describe what is happening in this situation. How will you solve this problem?

Vocabulary	**Spanish Cognates**
Tier 3	
• rational numbers	numeros racionales
• four operations	operaciónes

Standards Connections

4.OA.3, 6.NS.3, 7.NS.1, 7.NS.2 → *7.NS.3*

7.NS.3 → 7.EE.3, 7.EE.4

7.NS.3 Examples:

Examples of Rational-Number Problems 7.NS.3▲

1. During a phone call, Melanie was told of the most recent transactions in her company's business account. There were deposits of $1,250 and $3,040.57, three withdrawals of $400 each, and the bank removed two separate $35 penalties to the account that resulted from the bank's errors. Based on this information, how much did the balance of the account change?

Solution: The deposits are considered positive changes to the account, the three withdrawals are considered negative changes, and the removal of two penalties of $35 each may be thought of as subtracting debits to the account. The total change to the balance could be represented in this way:

$$\$1{,}250.00 + \$3{,}040.57 - 3(\$400.00) - 2(-\$35.00) = \$3{,}160.57.$$

Thus, the balance of the account increased by $3,160.57.

2. Find the product $(-373) \cdot 8$.

Solution: "I know that the first number has a factor of (–1) in it, so the product will be negative. Then I just need to find $373 \cdot 8 = 2400 + 560 + 24 = 2984$. So $(-373) \cdot 8 = -2984$."

3. Find the quotient $\left(-\frac{25}{28}\right) \div \left(-\frac{5}{4}\right)$.

Solution: "I know that the result is a positive number. This looks like a problem where I can divide the numerator and denominator: $\frac{25}{28} \div \frac{5}{4} = \frac{25 \div 5}{28 \div 4} = \frac{5}{7}$. The quotient is $\frac{5}{7}$."

4. Represent each of the following problems with a diagram, a number line, and an equation, and solve each problem.

   (a) A weather balloon is 100,000 feet above sea level, and a submarine is 3 miles below sea level, directly under the weather balloon. How far apart are the submarine and the weather balloon?

   (b) John was $3.75 in debt, and Mary had $0.50. John found some money in his old jacket and paid his debt. Afterward, he and Mary had the same amount of money. How much money was in John's jacket?

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

[1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

7th Grade – CCSS for Mathematics

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7.NS.A.3

Standard Explanation

Students solve real-world and mathematical problems involving positive and negative rational numbers while learning to compute sums, differences, products, and quotients of rational numbers. They also come to understand that every rational number can be written as a decimal with an expansion that eventually repeats or terminates (i.e., eventually repeats with zeros [7.NS.2c-d, 7.NS.3▲] [MP.1, MP.2, MP.5, MP.6, MP.7, MP.8] [CA *Mathematics Framework*, adopted Nov. 6, 2013]).

7.NS.3 Examples:

Example 1:
Calculate: [-10(-0.9)] – [(-10) • 0.11]

Example 2:
Jim's cell phone bill is automatically deducting $32 from his bank account every month. How much will the deductions total for the year?

Example 4:
A newspaper reports these changes in the price of a stock over four days: $\frac{-1}{8}$, $\frac{-5}{8}$, $\frac{3}{8}$, $\frac{-9}{8}$. What is the average
(North Carolina Unpacking Document, October 2012)

7.NS.3 Illustrative Tasks:

- Products and Quotients of Signed Rational Numbers, https://www.illustrativemathematics.org/content-standards/7/NS/A/3/tasks/1602

A water well drilling rig has dug to a height of -60 feet after one full day of continuous use.

a. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

b. If the rig has been running constantly and is currently at a height of -143.6 feet, for how long has the rig been running?

- Sharing Prize Money, https://www.illustrativemathematics.org/illustrations/298

The three seventh grade classes at Sunview Middle School collected the most boxtops for a school fundraiser, and so they won a $600 prize to share among them. Mr. Aceves' class collected 3,760 box tops, Mrs. Baca's class collected 2,301, and Mr. Canyon's class collected 1,855. How should they divide the money so that each class gets the same fraction of the prize money as the fraction of the box tops that they collected?

7th Grade – CCSS for Mathematics

7.NS.A.3

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7th Grade – CCSS for Mathematics

7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Essential Skills and Concepts:

- ☐ Use ratios and proportions to solve scale drawing problems
- ☐ Determine if shapes are similar or congruent
- ☐ Calculate the side lengths and angle measures

Question Stems and Prompts:

- ✓ Use the scale to determine the actual distance.
- ✓ Are the shapes similar or congruent? How do you know?
- ✓ How do you find the missing angle measure when two shapes are similar?

Math Vocabulary Tier 3	**Spanish Cognates**
• geometric	geométrico
• figures	figura
• length	longitud
• areas	área
• scale	escala

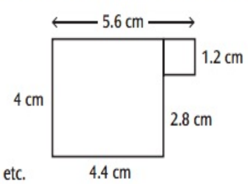
Standards Connections

6.G.1, 7.RP.2 → *7.G.1*

7.G.1 Examples:

Examples: Problems Involving Scale Drawings 7.G.1

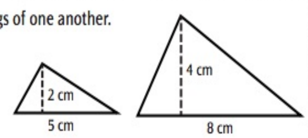
1. Julie shows you a scale drawing of her room. If 2 centimeters on the scale drawing equal 5 feet, what are the actual dimensions of Julie's room?



Solution: Since 2 centimeters in the drawing represent 5 feet, the conversion rate is $\frac{5}{2}$ ft/cm. So each measurement given in centimeters is multiplied to obtain the true measurement of the room in feet. Thus:

$5.6 \text{ cm} \rightarrow 5.6 \cdot \frac{5}{2} = 14 \text{ ft}$, $1.2 \text{ cm} \rightarrow 1.2 \cdot \frac{5}{2} = 3 \text{ ft}$, $2.8 \text{ cm} \rightarrow 2.8 \cdot \frac{5}{2} = 7 \text{ ft}$, etc.

2. Explain why the two triangles shown are *not* scale drawings of one another.



Solution: Since the ratios of the heights to bases of the triangles are different, one drawing cannot be a scale drawing of the other: $\frac{2}{5} \neq \frac{4}{8}$.

Adapted from ADE 2010 and NCDPI 2012.

7.G.1 Illustrative Task:

- Floor Plan
  https://www.illustrativemathematics.org/content-standards/7/G/A/1/tasks/107

Mariko has an $80 : 1$ scale-drawing of the floor plan of her house. On the floor plan, the dimensions of her rectangular living room are $1\frac{7}{8}$ inches by $2\frac{1}{2}$ inches.

What is the area of her real living room in square feet?

7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.

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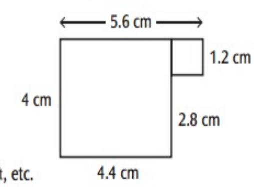

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$5.6 \text{ cm} \rightarrow 5.6 \cdot \frac{5}{2} = 14 \text{ ft}$, $1.2 \text{ cm} \rightarrow 1.2 \cdot \frac{5}{2} = 3 \text{ ft}$, $2.8 \text{ cm} \rightarrow 2.8 \cdot \frac{5}{2} = 7 \text{ ft}$, etc.

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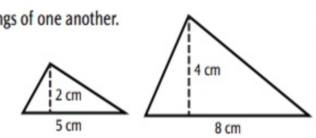

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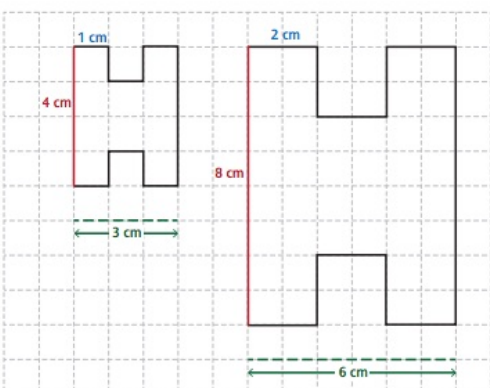
7.G.A.1

Standard Explanation

In grade seven, a critical area of instruction is for students to extend their study of geometry as they solve problems involving scale drawings and informal geometric constructions. Students also work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.

Standard 7.G.1 lays the foundation for students to understand dilations as geometric transformations. This will lead to a definition of the concept of similar shapes in eighth grade: shapes that can be obtained from one another through dilation. It is critical for students to grasp these ideas, as students need this comprehension to understand the derivation of the equations $\boldsymbol{y = mx}$ and $\boldsymbol{y = mx + b}$ by using similar triangles and the relationships between them. Thus standard 7.G.1 should be given significant attention in grade seven. Students solve problems involving scale drawings by applying their understanding of ratios and proportions, which started in grade six and continues in the grade-seven domain Ratios and Proportional Relationships (7.RP.1–3▲).

Teachers should note that the notion of *similarity* has not yet been addressed. Attempts to define similar shapes as those that have the "same shape but not necessarily the same size" should be avoided. Similarity will be defined precisely in grade eight, and imprecise notions of similarity may detract from student understanding of this important concept. Shapes drawn to scale are indeed similar to each other, but could safely be referred to as "scale drawings of each other" at this grade level. The concept of a scale drawing may be effectively introduced by allowing students to blow up or shrink pictures on grid paper.



By recording measurements in many examples, students come to see there are two important ratios with scale drawings: the ratios between two figures and the ratios within a single figure. Students should exploit these relationships when solving problems involving scale drawings, including problems that require mathematical justifications when drawings are not to scale (CA *Mathematics Framework*, adopted Nov. 6, 2013).

7th Grade – CCSS for Mathematics

7.G.A.1

Standard Explanation

In grade seven, a critical area of instruction is for students to extend their study of geometry as they solve problems involving scale drawings and informal geometric constructions. Students also work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.

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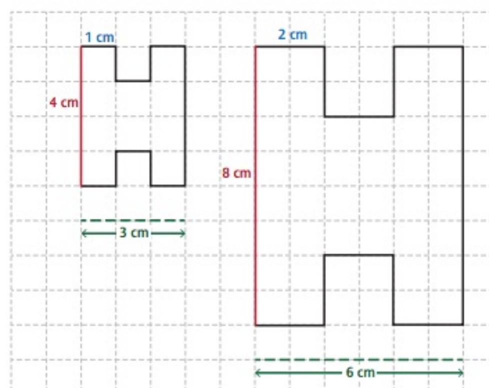

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7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Essential Skills and Concepts:
- ☐ Draw geometric shapes from given conditions
- ☐ Draw triangles from given conditions

Question Stems and Prompts:
- ✓ What do the angles tell you about a triangle?
- ✓ What do side lengths tell you about a shape?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• geometric	geométrico
• triangles	triángulo
• sides	
• angles	ángulos

Standards Connections
7.G.2

7th Grade – CCSS for Mathematics

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Standards Connections
7.G.2

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7th Grade – CCSS for Mathematics

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Standard Explanation
Students draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions, focusing on triangles (7.G.2). They work with three-dimensional figures and relate them to two-dimensional figures by examining cross-sections that result when three-dimensional figures are split (7.G.3). Students also describe how two or more objects are related in space (e.g., skewed lines and the possible ways in which three planes might intersect).

SBAC Sample Items:

Example Stem: A triangle has a 45º angle, a 60º angle, and a side 3 centimeters in length.

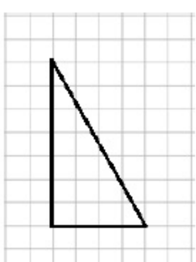
Select True or False for each statement about this type of triangle.

Statement	True	False
The triangle might be an isosceles triangle.		
The triangle must be an acute triangle.		
The triangle must contain an angle measuring 75º.		

Example Stem: Use the Connect Line tool to draw a triangle with a 90º angle, a side with a length of 7 units, and a side with a length of 4 units. Each square on the grid is 1 unit in length.

Interaction: The student is given the Connect Line, Add Point, and Delete tools to generate line segments on a grid.

Rubric: (1 point) The student correctly constructs the figure described.



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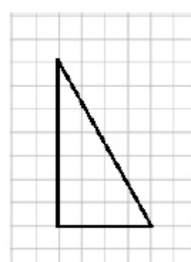

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7.G.A Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Essential Skills and Concepts:
- ☐ Cut three-dimensional figures
- ☐ Describe two-dimensional made from three-dimensional figures

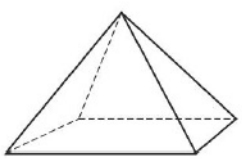
Question Stems and Prompts:
- ✓ What figure is made when the three-dimensional figure is cut?
- ✓ What attributes make up the figures?

Math Vocabulary	Spanish Cognates
Tier 3	
• figure	figura
• plane	plano
• prisms	prisma
• pyramids	pirámide
• rectangular	rectangular

Standards Connections
7.G.3

SBAC Sample Item:

Example Stem: This figure is a square pyramid.



Select **all** figures that can be formed by a vertical slice perpendicular to the base of the square pyramid.

A. Isosceles Trapezoid
B. Line segment
C. Square
D. Triangle

7th Grade – CCSS for Mathematics

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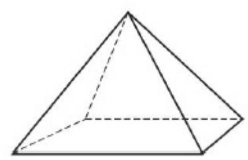
Standards Connections
7.G.3

SBAC Sample Item:

Example Stem: This figure is a square pyramid.



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7.G.A.3

Standard Explanation

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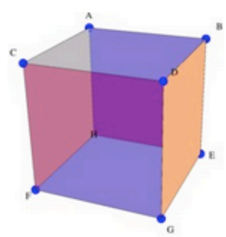
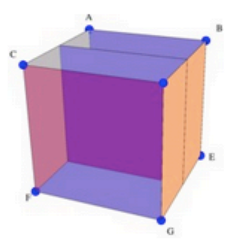
7.G.3 Illustrative Task:

- Cube Ninjas!
  https://www.illustrativemathematics.org/content-standards/7/G/A/3/tasks/1532

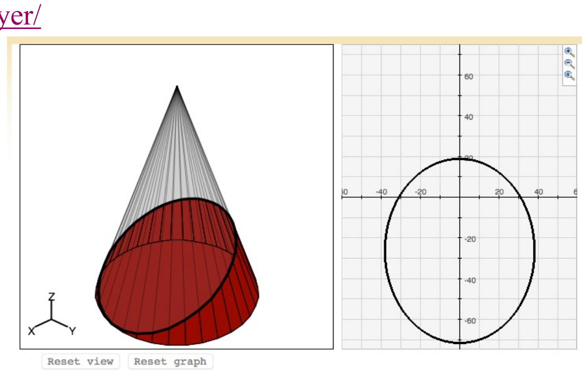
Imagine you are a ninja that can slice solid objects straight through. You have a solid cube in front of you. You are curious about what 2-dimensional shapes are formed when you slice the cube. For example, if you make a slice through the center of the cube that is parallel to one of the faces, the cross section is a square:





Other Resources:

- Cross Section Flyer
  http://www.shodor.org/interactivate/activities/CrossSectionFlyer/



7th Grade – CCSS for Mathematics

7.G.A.3

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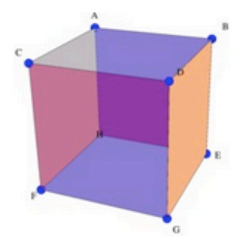

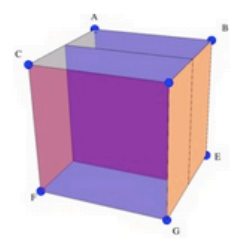

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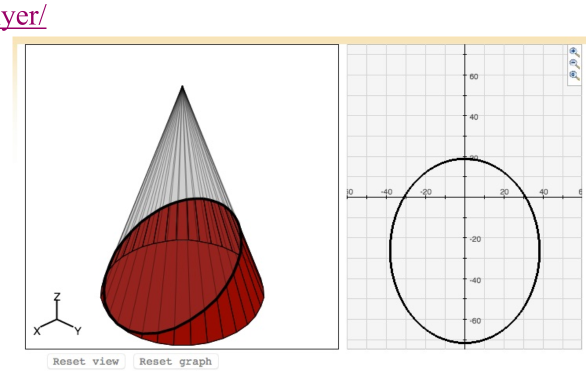

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7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Essential Skills and Concepts:
- ☐ Solve real-life and mathematical problems for the circumference and area of circles
- ☐ Understand the relationship between circumference and area of a circle
- ☐ Use the formula for area of a circle to solve problems
- ☐ Use the formula for circumference of a circle to solve problems

Question Stems and Prompts:
- ✓ What is the relationship between area and circumference of a circle?
- ✓ What is the relationship between the radius and circumference?
- ✓ What are the formulas for the area and circumference of a circle? Describe a real-life situation when you would use each of these formulas.
- ✓ What does your solution mean in the context of the problem?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• circumference	circunferencia
• area	área
• radius	radio

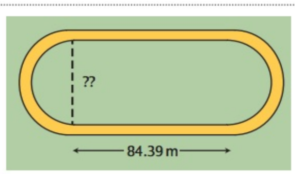
Standards Connections
6.G.1 → *7.G.4*

7.G.4 Examples:

Examples: Working with the Circumference and Area of a Circle 7.G.4

1. Students can explore the relationship between the circumference of a circle and its diameter (or radius). For example, by tracing the circumference of a cylindrical can of beans or some other cylinder on patty paper or tracing paper and finding the diameter by folding the patty paper appropriately, students can find the approximate diameter of the base of the cylinder. If they measure a piece of string the same length as the diameter, they will find that the string can wrap around the can approximately three and one-sixth times. That is, they find that $C \approx 3\frac{1}{6} \cdot d \approx 3.16$. When students do this for a variety of objects, they start to see that the ratio of the circumference of a circle to its diameter is always approximately the same number (π).


2. The total length of a standard track is 400 meters. The straight sides of the track each measure 84.39 meters. Assuming the rounded sides of the track are half-circles, find the distance from one side of the track to the other.



Solution: Together, the two rounded portions of the track make one circle, the circumference of which is $400 - 2(84.39) = 231.22$ meters. The length across the track is represented by the diameter of this circle. If the diameter is labeled d, then the resulting equation is $231.22 = \pi d$. Using a calculator and an approximation for π as 3.14, students arrive at $d = 231.22 \div \pi \approx 231.22 \div 3.14 \approx 73.64$ meters.

Adapted from ADE 2010.

7th Grade – CCSS for Mathematics

7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Essential Skills and Concepts:
☐ Solve real-life and mathematical problems for the circumference and area of circles
☐ Understand the relationship between circumference and area of a circle
☐ Use the formula for area of a circle to solve problems
☐ Use the formula for circumference of a circle to solve problems

Question Stems and Prompts:
✓ What is the relationship between area and circumference of a circle?
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Math Vocabulary / Spanish Cognates
Tier 3
• circumference circunferencia
• area área
• radius radio

Standards Connections
6.G.1 → 7.G.4

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Examples: Working with the Circumference and Area of a Circle 7.G.4

1. Students can explore the relationship between the circumference of a circle and its diameter (or radius). For example, by tracing the circumference of a cylindrical can of beans or some other cylinder on patty paper or tracing paper and finding the diameter by folding the patty paper appropriately, students can find the approximate diameter of the base of the cylinder. If they measure a piece of string the same length as the diameter, they will find that the string can wrap around the can approximately three and one-sixth times. That is, they find that $C \approx 3\frac{1}{6} \cdot d \approx 3.16$. When students do this for a variety of objects, they start to see that the ratio of the circumference of a circle to its diameter is always approximately the same number (π).


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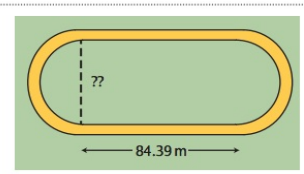

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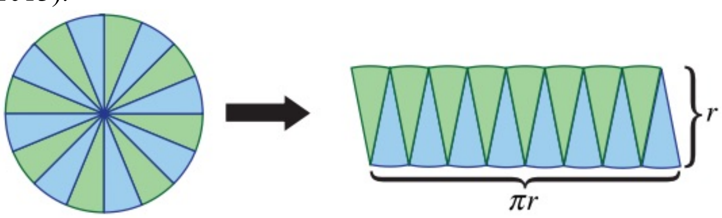
7th Grade – CCSS for Mathematics

7.G.B.4

Standard Explanation

In grade seven, students know the formulas for the area and circumference of a circle and use them to solve problems (7.G.4). To "know the formula" means to have an understanding of why the formula works and how the formula relates to the measure (area and circumference) and the figure. For instance, students can cut circles into finer and finer pie pieces (sectors) and arrange them into a shape that begins to approximate a parallelogram. Because of the way the shape was created, it has a length of approximately $\boldsymbol{\pi r}$ and a height of approximately $\boldsymbol{r}$. Therefore, the approximate area of this shape is $\boldsymbol{\pi r^2}$, which informally justifies the formula for the area of a circle (CA *Mathematics Framework*, adopted Nov. 6, 2013).

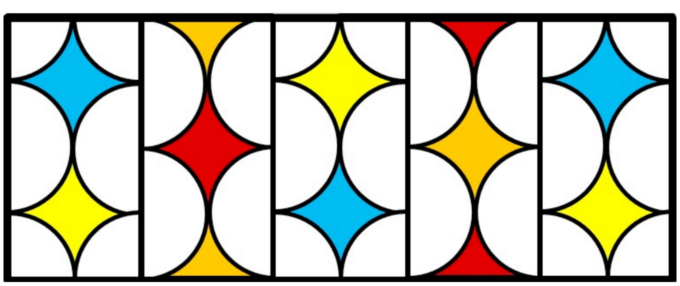


Adapted from KATM 2012, 7th Grade Flipbook.

7.G.4 Illustrative Task:

- Stained Glass https://www.illustrativemathematics.org/content-standards/7/G/B/4/tasks/1513

The students in Mr. Rivera's art class are designing a stained-glass window to hang in the school entryway. The window will be 2 feet tall and 5 feet wide. They have drawn the design below:



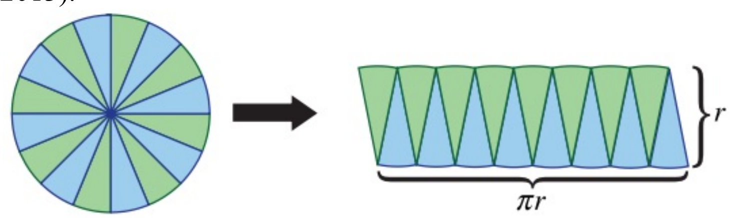
They have raised $100 for the materials for the project. The colored glass costs $5 per square foot and the clear glass costs $3 per square foot. The materials they need to join the pieces of glass together costs 10 cents per foot and the frame costs $4 per foot.

Do they have enough money to cover the costs of the materials they will need to make the window?

7.G.B.4

Standard Explanation

In grade seven, students know the formulas for the area and circumference of a circle and use them to solve problems (7.G.4). To "know the formula" means to have an understanding of why the formula works and how the formula relates to the measure (area and circumference) and the figure. For instance, students can cut circles into finer and finer pie pieces (sectors) and arrange them into a shape that begins to approximate a parallelogram. Because of the way the shape was created, it has a length of approximately $\boldsymbol{\pi r}$ and a height of approximately $\boldsymbol{r}$. Therefore, the approximate area of this shape is $\boldsymbol{\pi r^2}$, which informally justifies the formula for the area of a circle (CA *Mathematics Framework*, adopted Nov. 6, 2013).



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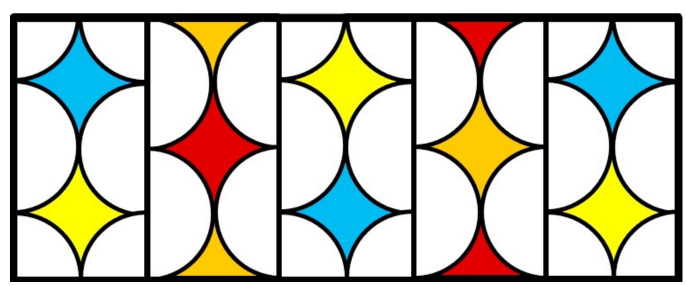

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7th Grade – CCSS for Mathematics

7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Essential Skills and Concepts:

- ☐ Solve real-life and mathematical problems involving angle measure, including multi-step problems
- ☐ Write and solve equations for angle measure problems
- ☐ Understand and use facts about supplementary, commentary, vertical, and adjacent angles to solve problems

Question Stems and Prompts:

- ✓ How did you solve for the missing angle?
- ✓ What did you know about angles that helped you solve this problem?
- ✓ What equation could you write to help you solve this problem?
- ✓ What do the parts of your equation represent in the context of the problem?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• supplementary	suplementario
• complementary	complementario
• vertical	vertical
• adjacent	adyacente

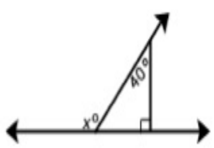
Standards Connections

4.MD.7 → *7.G.5*

7.G.5 Examples:

Example1:

Write and solve an equation to find the measure of angle *x*.



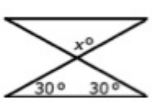
Solution:

Find the measure of the missing angle inside the triangle (180 – 90 – 40), or 50°.

The measure of angle *x* is supplementary to 50°, so subtract 50 from 180 to get a measure of 130° for *x*.

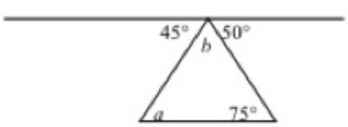
Example 2:

Find the measure of angle *x*.



Example 3:

Find the measure of angle *b*.



Note: Not drawn to scale.

7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Essential Skills and Concepts:

- ☐ Solve real-life and mathematical problems involving angle measure, including multi-step problems
- ☐ Write and solve equations for angle measure problems
- ☐ Understand and use facts about supplementary, commentary, vertical, and adjacent angles to solve problems

Question Stems and Prompts:

- ✓ How did you solve for the missing angle?
- ✓ What did you know about angles that helped you solve this problem?
- ✓ What equation could you write to help you solve this problem?
- ✓ What do the parts of your equation represent in the context of the problem?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• supplementary	suplementario
• complementary	complementario
• vertical	vertical
• adjacent	adyacente

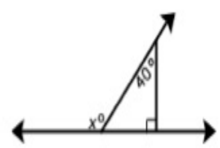
Standards Connections

4.MD.7 → *7.G.5*

7.G.5 Examples:

Example1:

Write and solve an equation to find the measure of angle *x*.



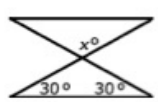
Solution:

Find the measure of the missing angle inside the triangle (180 – 90 – 40), or 50°.

The measure of angle *x* is supplementary to 50°, so subtract 50 from 180 to get a measure of 130° for *x*.

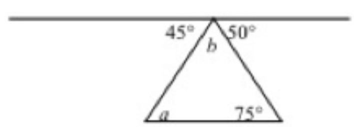
Example 2:

Find the measure of angle *x*.



Example 3:

Find the measure of angle *b*.



Note: Not drawn to scale.

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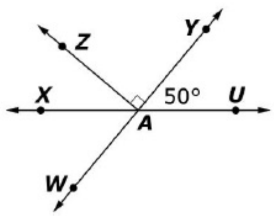
7.G.B Solve real-life and mathematical problems. involving angle measure, area, surface area, and volume.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Standard Explanation
Students use understandings of angles and deductive reasoning to write and solve equations (North Carolina Department of Public Instruction, 2012).

SBAC Sample Items:

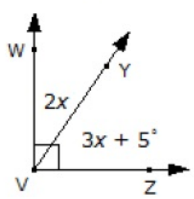
Example Stem: Lines *XU* and *WY* intersect at point *A*.



Based on the diagram, determine whether each statement is true. Select True or False for each statement.

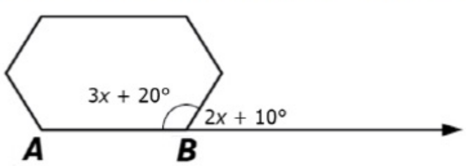
Statement	True	False
$m\angle XAZ = 180° - m\angle ZAY - m\angle YAU$		
$m\angle WAZ = m\angle WAY - m\angle ZAY$		
$m\angle WAU = m\angle XAZ - m\angle ZAY$		

Example Stem: Consider this figure.



Enter the measure of $\angle YVZ$, in degrees.

Example Stem: The base of a hexagon lies on ray *AB* as shown.



Based on the diagram, determine whether each equation is true. Select True or False for each statement.

Statement	True	False
$3x + 20° = 110°$		
$2x + 10° = 70°$		
$5x + 30° = 90°$		

7th Grade – CCSS for Mathematics

7.G.B Solve real-life and mathematical problems. involving angle measure, area, surface area, and volume.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

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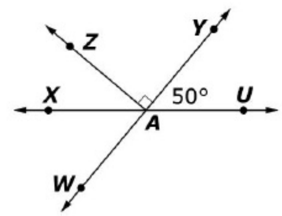

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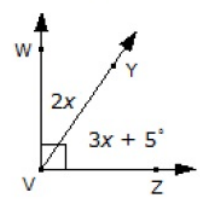

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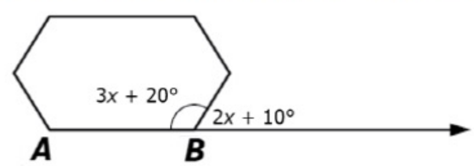

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7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Essential Skills and Concepts:
- ☐ Solve real-life and mathematical problems for the area of two-dimensional objects composed of polygons
- ☐ Solve real-life and mathematical problems for the volume of three-dimensional objects composed of prisms and cubes

Question Stems and Prompts:
- ✓ Would you find the area, surface area, or volume to solve this problem? Justify your thinking.
- ✓ The floor plan for a bedroom is composed of 3 rectangles in a u-shape. Describe how you would determine the amount of carpet in square feet that you would need to buy to cover the floor of the bedroom.
- ✓ How do you find the volume of a 3D figure composed of prisms and cubes?
- ✓ Compare the area of these two figures. Which figure has the greater area? How do you know?
- ✓ If you needed to buy paint to cover a box, what would you calculate to figure out how much you need to buy? Why?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• area	área
• volume	volumen
• surface area	área de superficie
• triangles	triángulo
• quadrilaterals	cuadrilátero
• polygons	poligono
• cubes	cubo
• prism	prisma

Standards Connections
6.G.1, 6.G.2, 6.G.4→ *7.G.6*

7.G.6 Examples:

Example: Surface Area and Volume	7.G.6

The surface area of a cube is 96 square inches. What is the volume of the cube?

Solution: Students understand from working with nets in grade six that the cube has six faces, all with equal area. Thus, the area of one face of the cube is $96 \div 6 = 16$ square inches. Since each face is a square, the length of one side of the cube is 4 inches. This makes the volume $V = 4^3 = 64$ cubic inches.

7th Grade – CCSS for Mathematics

7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

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Math Vocabulary	**Spanish Cognates**
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7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Standard Explanation

Students continue work from grades five and six to solve problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms (7.G.6) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

7.G.6 Illustrative Task:

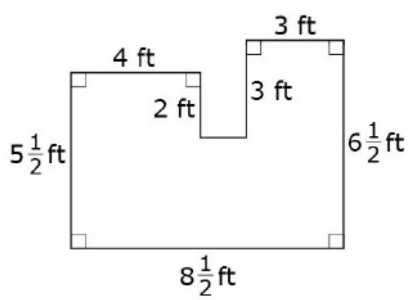
- Sand Under the Swing Set
  https://www.illustrativemathematics.org/content-standards/7/G/B/6/tasks/266

The 7th graders at Sunview Middle School were helping to renovate a playground for the kindergartners at a nearby elementary school. City regulations require that the sand underneath the swings be at least 15 inches deep. The sand under both swing sets was only 12 inches deep when they started.

The rectangular area under the small swing set measures 9 feet by 12 feet and required 40 bags of sand to increase the depth by 3 inches. How many bags of sand will the students need to cover the rectangular area under the large swing set if it is 1.5 times as long and 1.5 times as wide as the area under the small swing set?

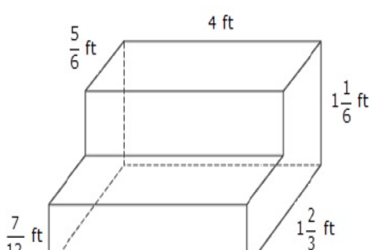
SBAC Sample Items:

Example Stem 1: This is the floor plan of Julie's bedroom.



Enter the amount of carpet, in square feet, needed to completely cover Julie's bedroom floor.

Example Stem 1: The figure shows a set of concrete stairs to be built.



Enter the amount of concrete, in cubic feet, needed to build the stairs. Round your answer to the nearest hundredth.

7th Grade – CCSS for Mathematics

7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Standard Explanation

Students continue work from grades five and six to solve problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms (7.G.6) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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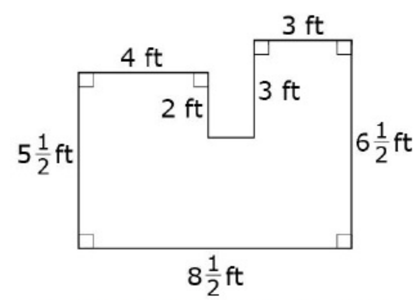

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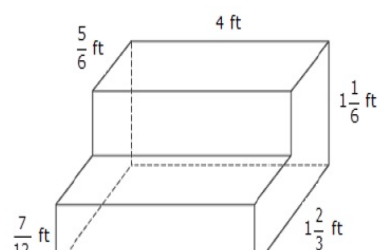

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7.EE.A Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Essential Skills and Concepts:
- ☐ Understand the properties of operations
- ☐ Add linear expressions
- ☐ Subtract linear expressions
- ☐ Factor linear expressions
- ☐ Understand rational coefficients

Question Stems and Prompts:
- ✓ How do you factor expressions? Explain the process
- ✓ What is the result when like terms are combined?

Vocabulary	Spanish Cognates
Tier 2	
• expand	expandir
Tier 3	
• linear	lineal
• factor	factor
• expressions	expresiones
• rational	racional
• coefficients	coeficientes

Standards Connections
6.EE.3, 6.EE.4 → *7.EE.1*
7.EE.1 – 7.EE.2

7.EE.1 Examples:

Focus, Coherence, and Rigor

The work in standards **7.EE.1–2▲** is closely connected to standards **7.EE.3–4▲**, as well as the multi-step proportional reasoning problems in the domain Ratios and Proportional Relationships (**7.RP.3▲**). Students' work with rational-number arithmetic (**7.NS▲**) is particularly relevant when they write and solve equations (**7.EE▲**). Procedural fluency in solving these types of equations is an explicit goal of standard **7.EE.4a**.

Examples: Working with Expressions.

1. A rectangle is twice as long as it is wide. Find as many different ways as you can to write an expression for the perimeter of such a rectangle.
   Solution: If W represents the width of the rectangle, and L represents the length, then we could express the perimeter as $L + W + L + W$. We could rewrite this as $2L + 2W$. Knowing that $L = 2W$, the perimeter could also be given by $W + W + 2W + 2W$, which we could rewrite as $6W$. Alternatively, we know that $W = \frac{L}{2}$, so the perimeter could be given in terms of the length as $L + L + \frac{L}{2} + \frac{L}{2}$, which we could rewrite as $3L$.
2. While Chris was driving a Canadian car, he figured out a way to mentally convert the outside temperature that the car displayed in degrees Celsius to degrees Fahrenheit. This was his method: "I take the temperature it shows and I double it, then I subtract one-tenth of that doubled amount. Then, I add 32 to get the Fahrenheit temperature." The standard expression for finding the temperature in degrees Fahrenheit when the degrees Celsius is known is given by $\frac{9}{5}C + 32$, where C is the temperature in degrees Celsius. Is Chris's method correct?

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

7th Grade – CCSS for Mathematics

7.EE.A Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Essential Skills and Concepts:
☐ Understand the properties of operations
☐ Add linear expressions
☐ Subtract linear expressions
☐ Factor linear expressions
☐ Understand rational coefficients

Question Stems and Prompts:
✓ How do you factor expressions? Explain the process
✓ What is the result when like terms are combined?

Vocabulary / Spanish Cognates
Tier 2
• expand — expandir
Tier 3
• linear — lineal
• factor — factor
• expressions — expresiones
• rational — racional
• coefficients — coeficientes

Standards Connections
6.EE.3, 6.EE.4 → 7.EE.1
7.EE.1 – 7.EE.2

7.EE.1 Examples:

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(CA Mathematics Framework, adopted Nov. 6, 2013)

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7.EE.A.1

Standard Explanation

In grade six, students began the study of equations and inequalities and methods for solving them. In grade seven, students build on this understanding and use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. Students also work toward fluently solving equations of the form $\boldsymbol{px+q=r}$ and $\boldsymbol{p\,(x+q)=r}$.

This cluster of standards calls for students to work with linear expressions where the distributive property plays a prominent role (7.EE.1▲). A fundamental understanding is that the distributive property works “on the right” as well as “on the left,” in addition to “forward” and “backward.” That is students should have opportunities to see that for numbers *a*, *b*, and *c* and *x*, *y*, and *z*.

$$a(b+c)=ab+ac \quad \text{and} \quad ab+ac=a(b+c)$$

$$(x+y)z=xz+yz \quad \text{and} \quad xz+yz=(x+y)z$$

(CA *Mathematics Framework*, adopted Nov. 6, 2013).

7.EE.1 Illustrative Tasks:

- Equivalent Expressions? https://www.illustrativemathematics.org/content-standards/7/EE/A/tasks/543

If we multiply $\frac{x}{2}+\frac{3}{4}$ by 4, we get $2x+3$. Is $2x+3$ an equivalent expression to $\frac{x}{2}+\frac{3}{4}$?

- Writing Expressions, https://www.illustrativemathematics.org/content-standards/7/EE/A/1/tasks/541

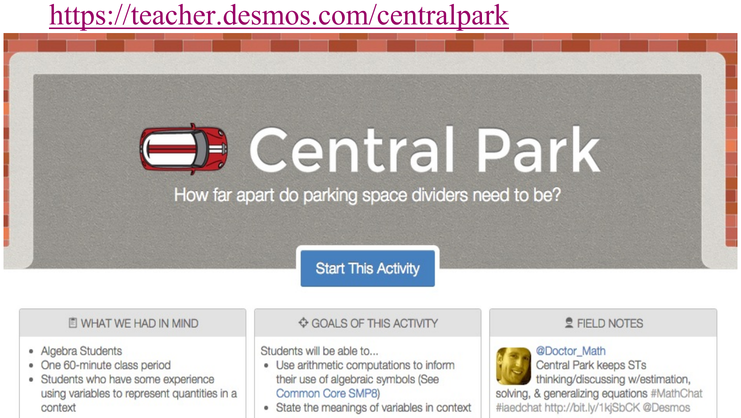
Write an expression for the sequence of operations.

a. Add 3 to x, subtract the result from 1, then double what you have.

b. Add 3 to x, double what you have, then subtract 1 from the result.

Other Resources:

- Teacher Desmos - Central Park https://teacher.desmos.com/centralpark



7th Grade – CCSS for Mathematics

7.EE.A.1

Standard Explanation

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$$a(b+c)=ab+ac \quad \text{and} \quad ab+ac=a(b+c)$$

$$(x+y)z=xz+yz \quad \text{and} \quad xz+yz=(x+y)z$$

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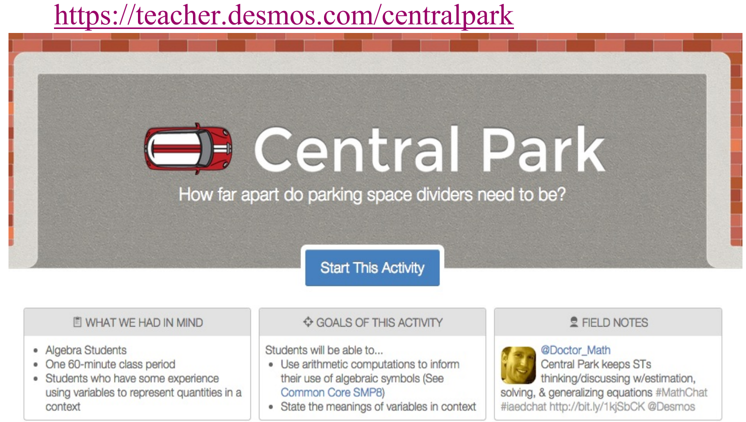

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7th Grade – CCSS for Mathematics

7.EE.A Use properties of operations to generate equivalent expressions.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Essential Skills and Concepts:

- ☐ Expanding expressions by rewriting them
- ☐ Identify equivalent expressions by using substitution

Question Stems and Prompts:

- ✓ Why are the expressions equivalent? Justify your answer
- ✓ Which expression represents the real world example?
- ✓ What is the word form of the expression?

Vocabulary	Spanish Cognates
Tier 2	
• rewrite	
• related	relacionado
• form	forma
Tier 3	
• expression	expresión
• quantities	cantidades

Standards Connections

7.EE.1 – *7.EE.2*

7.EE.2 Examples:

Examples: Working with Expressions 7.EE.2▲

1. A rectangle is twice as long as it is wide. Find as many different ways as you can to write an expression for the perimeter of such a rectangle.

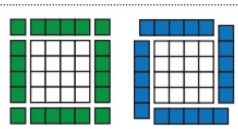
Solution: If W represents the width of the rectangle and L represents the length, then the perimeter could be expressed as $L+W+L+W$. This could be rewritten as $2L+2W$. If it is known that $L=2W$, the perimeter could be represented by $W+W+2W+2W$, which could be rewritten as $6W$. Alternatively, if $W=\frac{L}{2}$, the perimeter could be given in terms of the length as $L+L+\frac{L}{2}+\frac{L}{2}$, which could be rewritten as $3L$.

Adapted from ADE 2010.

2. While Chris was driving a Canadian car, he figured out a way to mentally convert the outside temperature that the car displayed in degrees Celsius to degrees Fahrenheit. This was his method: "I took the temperature it showed and doubled it. Then I subtracted one-tenth of that doubled amount. Finally, I added 32 to get the Fahrenheit temperature." The standard expression for finding the temperature in degrees Fahrenheit when the Celsius reading is known is $\frac{9}{5}C+32$, where C is the temperature in degrees Celsius. Was Chris's method correct?

Solution: If C is the temperature in degrees Celsius, then the first step in Chris's calculation was to find $2C$. Then, he subtracted one-tenth of that quantity, which yielded $\frac{1}{10}(2C)$. Finally, he added 32. The resulting expression was $2C-\frac{1}{10}(2C)+32$. This could be rewritten as $2C-\frac{1}{5}C+32$. Combining the first two terms, we got $2C-\frac{1}{5}C+32=\left(2-\frac{1}{5}\right)C+32=\left(\frac{10}{5}-\frac{1}{5}\right)C+32=\frac{9}{5}C+32$. Chris's calculation was correct.

3. In the well-known "Pool Border Problem," students are asked to determine the number of tiles needed to construct a border for a pool (or grid) of size $n \times n$, represented by the white tiles in the figure. Students may first examine several examples and organize their counting of the border tiles, after which they can be asked to develop an expression for the number of border tiles, B (**MP.8**). Many different expressions are correct, all of which are equivalent to $4n+4$. However, different expressions arise from different ways of seeing the construction of the border. A student who sees the border as four sides of length n plus four corners might develop the expression $4n+4$, while a student who sees the border as four sides of length $n+1$ may find the expression $4(n+1)$. It is important for students to see many different representations and understand that these representations express the same quantity in different ways (**MP.7**).



$B=4n+4$ $B=4(n+1)$

Adapted from NCDPI 2013b.

7.EE.A Use properties of operations to generate equivalent expressions.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Essential Skills and Concepts:

- ☐ Expanding expressions by rewriting them
- ☐ Identify equivalent expressions by using substitution

Question Stems and Prompts:

- ✓ Why are the expressions equivalent? Justify your answer
- ✓ Which expression represents the real world example?
- ✓ What is the word form of the expression?

Vocabulary	Spanish Cognates
Tier 2	
• rewrite	
• related	relacionado
• form	forma
Tier 3	
• expression	expresión
• quantities	cantidades

Standards Connections

7.EE.1 – *7.EE.2*

7.EE.2 Examples:

Examples: Working with Expressions 7.EE.2▲

1. A rectangle is twice as long as it is wide. Find as many different ways as you can to write an expression for the perimeter of such a rectangle.

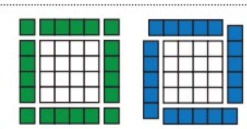
Solution: If W represents the width of the rectangle and L represents the length, then the perimeter could be expressed as $L+W+L+W$. This could be rewritten as $2L+2W$. If it is known that $L=2W$, the perimeter could be represented by $W+W+2W+2W$, which could be rewritten as $6W$. Alternatively, if $W=\frac{L}{2}$, the perimeter could be given in terms of the length as $L+L+\frac{L}{2}+\frac{L}{2}$, which could be rewritten as $3L$.

Adapted from ADE 2010.

2. While Chris was driving a Canadian car, he figured out a way to mentally convert the outside temperature that the car displayed in degrees Celsius to degrees Fahrenheit. This was his method: "I took the temperature it showed and doubled it. Then I subtracted one-tenth of that doubled amount. Finally, I added 32 to get the Fahrenheit temperature." The standard expression for finding the temperature in degrees Fahrenheit when the Celsius reading is known is $\frac{9}{5}C+32$, where C is the temperature in degrees Celsius. Was Chris's method correct?

Solution: If C is the temperature in degrees Celsius, then the first step in Chris's calculation was to find $2C$. Then, he subtracted one-tenth of that quantity, which yielded $\frac{1}{10}(2C)$. Finally, he added 32. The resulting expression was $2C-\frac{1}{10}(2C)+32$. This could be rewritten as $2C-\frac{1}{5}C+32$. Combining the first two terms, we got $2C-\frac{1}{5}C+32=\left(2-\frac{1}{5}\right)C+32=\left(\frac{10}{5}-\frac{1}{5}\right)C+32=\frac{9}{5}C+32$. Chris's calculation was correct.

3. In the well-known "Pool Border Problem," students are asked to determine the number of tiles needed to construct a border for a pool (or grid) of size $n \times n$, represented by the white tiles in the figure. Students may first examine several examples and organize their counting of the border tiles, after which they can be asked to develop an expression for the number of border tiles, B (**MP.8**). Many different expressions are correct, all of which are equivalent to $4n+4$. However, different expressions arise from different ways of seeing the construction of the border. A student who sees the border as four sides of length n plus four corners might develop the expression $4n+4$, while a student who sees the border as four sides of length $n+1$ may find the expression $4(n+1)$. It is important for students to see many different representations and understand that these representations express the same quantity in different ways (**MP.7**).



$B=4n+4$ $B=4(n+1)$

Adapted from NCDPI 2013b.

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7.EE.A Use properties of operations to generate equivalent expressions.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Standard Explanation

Students combine their understanding of parentheses as denoting single quantities with the standard order of operations, operations with rational numbers, and the properties above to rewrite expressions in different ways (7.EE.2▲).

Note that the standards do not expressly refer to "simplifying" expressions. Simplifying an expression is a special case of generating equivalent expressions. This is not to say that simplifying is never important, but whether one expression is "simpler" than another to work with often depends on the context. For example, the expression $\mathbf{50 + (x - 500) \cdot 0.20}$ represents the cost of a phone plan wherein the base cost is \$50 and any minutes over 500 cost \$0.20 per minute (valid for $\mathbf{x \geq 500}$). However, it is more difficult to see how the equivalent expression $\mathbf{0.20x - 50}$ also represents the cost of this phone plan.

As students become familiar with multiple ways of writing an expression, they also learn that different ways of writing expressions can serve varied purposes and provide different ways of seeing a problem. In example 3 below, the connection between the expressions and the figure emphasizes that both represent the same number, and the connection between the structure of each expression and a method of calculation emphasizes the fact that expressions are built from operations on numbers (adapted from UA Progressions Documents 2011d) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

Common Misconceptions: Working with the Distributive Property 7.EE.2▲

Students see expressions like $7-2(8-1.5x)$ and realize that the expression $(8-1.5x)$ is treated as a separate quantity in its own right, being multiplied by 2 and the result being subtracted from 7 (**MP.7**). Students may mistakenly come up with the expressions below, and each case offers a chance for class discussion about why it is not equivalent to the original (**MP.3**):

- $5(8-1.5x)$, subtracting $7-2$ without realizing the multiplication must be done first
- $7-2(6.5x)$, erroneously combining 8 and $-1.5x$ by neglecting to realize that these are not like terms
- $7-16-3x$, by misapplying the distributive property or not being attentive to the rules for multiplying negative numbers

Students should have the opportunity to see this expression as equivalent to both $7+(-2)(8-1.5x)$ and $7-(-2(8-1.5x))$, which can aid in seeing the correct way to handle the -2 part of the expression.

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

7th Grade – CCSS for Mathematics

7.EE.A Use properties of operations to generate equivalent expressions.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Standard Explanation

Students combine their understanding of parentheses as denoting single quantities with the standard order of operations, operations with rational numbers, and the properties above to rewrite expressions in different ways (7.EE.2▲).

Note that the standards do not expressly refer to "simplifying" expressions. Simplifying an expression is a special case of generating equivalent expressions. This is not to say that simplifying is never important, but whether one expression is "simpler" than another to work with often depends on the context. For example, the expression $\mathbf{50 + (x - 500) \cdot 0.20}$ represents the cost of a phone plan wherein the base cost is \$50 and any minutes over 500 cost \$0.20 per minute (valid for $\mathbf{x \geq 500}$). However, it is more difficult to see how the equivalent expression $\mathbf{0.20x - 50}$ also represents the cost of this phone plan.

As students become familiar with multiple ways of writing an expression, they also learn that different ways of writing expressions can serve varied purposes and provide different ways of seeing a problem. In example 3 below, the connection between the expressions and the figure emphasizes that both represent the same number, and the connection between the structure of each expression and a method of calculation emphasizes the fact that expressions are built from operations on numbers (adapted from UA Progressions Documents 2011d) (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

Essential Skills and Concepts:
- ☐ Solve positive and negative rational number problems
- ☐ Solve multi-step positive and negative decimal problems
- ☐ Solve multi-step positive and negative fraction problems
- ☐ Solve problems using order of operations

Question Stems and Prompts:
- ✓ What is a good method to use to check the reasonableness of the answer?
- ✓ What is the difference between fractions and decimals? What are similarities between fractions and decimals?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• positive	positivo
• negative	negativo
• fractions	fracción
• decimals	decimales

Standards Connections
7.NS.3 → *7.EE.3*

7.EE.3-4 Examples:

Examples: Solving Equations and Inequalities 7.EE.3–4▲ (MP.2, MP.4, MP. 7)

1. The youth group is going on a trip to the state fair. The trip costs $52.50 per student. Included in that price is $11.25 for a concert ticket and the cost of 3 passes, 2 for rides and 1 for game booths. Each of the passes costs the same price. Write an equation representing the cost of the trip, and determine the price of 1 pass.

Solution: Students can represent the situation with a tape diagram, showing that $3p+11.25$ represents the total cost of the trip if p represents the price of each pass. Students find the equation $3p+11.25=52.50$. They see the expression on the left side of the equation as some quantity $(3p)$ plus 11.25 equaling 52.50.

$52.50			
p	p	p	$11.25

In that case, the equation $52.50-11.25=41.25$ represents that quantity, by the relationship between addition and subtraction. So $3p=41.25$, which means that $p=41.25\div 3=13.75$. Thus, each pass costs $13.75.

(CA *Mathematics Framework*, adopted Nov. 6, 2013)

7th Grade – CCSS for Mathematics

7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

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Standards Connections
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7.EE.B.3

Standard Explanation

By grade seven, students begin to see whole numbers and their opposites, as well as positive and negative fractions, as belonging to a single system of rational numbers. Students solve multi-step problems involving rational numbers presented in various forms (whole numbers, fractions, and decimals), assessing the reasonableness of their answers (MP.1), and they solve problems that result in basic linear equations and inequalities (7.EE.3–4▲). This work is the culmination of many progressions of learning in arithmetic, problem solving, and mathematical practices (CA *Mathematics Framework*, adopted Nov. 6, 2013).

7.EE.3 Illustrative Tasks:

- Shrinking
  https://www.illustrativemathematics.org/content-standards/7/EE/B/3/tasks/108

When working on a report for class, Catrina read that a woman over the age of 40 can lose approximately 0.06 centimeters of height per year.

a. Catrina's aunt Nancy is 40 years old and is 5 feet 7 inches tall. Assuming her height decreases at this rate after the age of 40, about how tall will she be at age 65? (Remember that 1 inch = 2.54 centimeters.)

b. Catrina's 90-year-old grandmother is 5 feet 1 inch tall. Assuming her grandmother's height has also decreased at this rate, about how tall was she at age 40? Explain your reasoning.

- Who is the Better Batter?
  https://www.illustrativemathematics.org/content-standards/7/EE/B/3/tasks/1588

Below is a table showing the number of hits and the number of times at bat for two Major League Baseball players during two different seasons:

Season	Derek Jeter	David Justice
1995	12 hits in 48 at bats	104 hits in 411 at bats
1996	183 hits in 582 at bats	45 hits in 140 at bats

A player's *batting average* is the fraction of times at bat when the player gets a hit.

a. For each season, find the players' batting averages. Who has the better batting average?

b. For the combined 1995 and 1996 seasons, find the players' batting averages. Who has the better batting average?

c. Are the answers to (a) and (b) consistent? Explain.

7th Grade – CCSS for Mathematics

7.EE.B.3

Standard Explanation

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7th Grade – CCSS for Mathematics

7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Essential Skills and Concepts:
- ☐ Solve ratio word problems
- ☐ Model and solve equations
- ☐ Solve one and two step equations
- ☐ Graph inequalities on the number line

Question Stems and Prompts:
- ✓ Compare the solutions of the equations. How are they similar and how are they different?
- ✓ What do the lines on the graph represent?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• equations	ecuación
• linear	lineal
• inequalities	desigualdad
• ratios	proporción
• proportions	proporción

Standards Connections
6.EE.6, 6.EE.7, 6.EE.8, 7.NS.3→ *7.EE.4*

7.EE.3-4 Examples:

3. Florencia can spend at most $60 on clothes. She wants to buy a pair of jeans for $22 and spend the rest on T-shirts. Each shirt costs $8. Write an inequality for the number of T-shirts she can purchase.

Solution: If t represents the number of T-shirts Florencia buys, then an expression for the total amount she spends on clothes is $8t + 22$, since each T-shirt costs $8. The term ***at most*** might be new to students, but it indicates that the amount Florencia spends must be less than or equal to $60. The inequality that results is $8t + 22 \leq 60$. Note that the symbol "≤" is used here to denote that the amount Florencia spends can be less than ***or*** equal to $60. This symbol should be introduced in grade seven.

Adapted from NCDPI 2013b.

7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

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b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

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Standards Connections
6.EE.6, 6.EE.7, 6.EE.8, 7.NS.3→ *7.EE.4*

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3. Florencia can spend at most $60 on clothes. She wants to buy a pair of jeans for $22 and spend the rest on T-shirts. Each shirt costs $8. Write an inequality for the number of T-shirts she can purchase.

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7th Grade – CCSS for Mathematics

7.EE.B.4

Standard Explanation

By grade seven, students begin to see whole numbers and their opposites, as well as positive and negative fractions, as belonging to a single system of rational numbers. Students solve multi-step problems involving rational numbers presented in various forms (whole numbers, fractions, and decimals), assessing the reasonableness of their answers (MP.1), and they solve problems that result in basic linear equations and inequalities (7.EE.3–4▲). This work is the culmination of many progressions of learning in arithmetic, problem solving, and mathematical practices (CA *Mathematics Framework*, adopted Nov. 6, 2013).
reverses the order of the comparison it represents.

Estimation Strategies for Assessing Reasonableness of Answers (MP.1, MP.5)

Below are a few examples of estimation strategies that students may use to evaluate the reasonableness of their answers:

- **Front-end estimation with adjusting** — using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts
- **Clustering around an average** — when the values are close together, an average value is selected and multiplied by the number of values to determine an estimate
- **Rounding and adjusting** — rounding down or rounding up and then adjusting the estimate based on how much the rounding affected the original values
- **Using friendly or compatible numbers such as factors** — fitting numbers together (e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000)
- **Using benchmark numbers that are easy to compute** — selecting close whole numbers for fractions or decimals to determine an estimate

Adapted from KATM 2012, 7th Grade Flipbook.

7.EE.4 Illustrative Tasks:

- Fishing Adventures 2
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/643

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs of gear for the boat plus 10 lbs of gear for each person.

a. Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.

b. Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? What is the maximum number of people that may rent a boat?

- Bookstore Account
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/1475

a. At the beginning of the month, Evan had $24 in his account at the school bookstore. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then represent each transaction on a number line. What is the unknown quantity in each case?

7.EE.B.4

Standard Explanation

By grade seven, students begin to see whole numbers and their opposites, as well as positive and negative fractions, as belonging to a single system of rational numbers. Students solve multi-step problems involving rational numbers presented in various forms (whole numbers, fractions, and decimals), assessing the reasonableness of their answers (MP.1), and they solve problems that result in basic linear equations and inequalities (7.EE.3–4▲). This work is the culmination of many progressions of learning in arithmetic, problem solving, and mathematical practices (CA *Mathematics Framework*, adopted Nov. 6, 2013).
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  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/643

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs of gear for the boat plus 10 lbs of gear for each person.

a. Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.

b. Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? What is the maximum number of people that may rent a boat?

- Bookstore Account
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/1475

a. At the beginning of the month, Evan had $24 in his account at the school bookstore. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then represent each transaction on a number line. What is the unknown quantity in each case?

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7.SP.A Use random sampling to draw inferences about a population.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Essential Skills and Concepts:
- ☐ Understand the basis of statistics
- ☐ Understand random sampling
- ☐ Understand how generalizations about a population can be made

Question Stems and Prompts:
- ✓ Is the sample representative of the population? Justify your answer.
- ✓ What sampling methods were used for the sample?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• population	poblacion
• sample	
• random	
• statistic	estadística

Standards Connections
6.SP.1, 6.SP.2, 7.SP.5 → *7.SP.1*
7.SP.1 → 7.SP.2

7.SP.1 Example:

Example: Random Sampling 7.SP.1

The table below shows data collected from two random samples of 100 students regarding their school lunch preferences. Make at least two inferences based on the results.

	Hamburgers	Tacos	Pizza	Total
Student Sample 1	12	14	74	100
Student Sample 2	12	11	77	100

Possible solutions: Since the sample sizes are relatively large, and a vast majority in both samples prefer pizza, it would be safe to draw these two conclusions:

1. Most students prefer pizza.
2. More students prefer pizza than hamburgers and tacos combined.

Adapted from ADE 2010.

7th Grade – CCSS for Mathematics

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Standard Explanation

Seventh-grade students use data from a random sample to draw inferences about a population with an unknown characteristic (7.SP.1–2). For example, students could predict the mean height of seventh-graders by collecting data in several random samples.

Students recognize that it is difficult to gather statistics on an entire population. They also learn that a random sample can be representative of the total population and will generate valid predictions. Students use this information to draw inferences from data (MP.1, MP.2, MP.3, MP.4, MP.5, MP.6, MP.7). The standards in the 7.SP.1–2 cluster represent opportunities to apply percentages and proportional reasoning. In order to make inferences about a population, one applies such reasoning to the sample and the entire population (CA *Mathematics Framework*, adopted Nov. 6, 2013).

7.SP.1 Illustrative Task:

- Mr. Brigg's Class Likes Math
  https://www.illustrativemathematics.org/content-standards/7/SP/A/1/tasks/974

In a poll of Mr. Briggs's math class, 67% of the students say that math is their favorite academic subject. The editor of the school paper is in the class, and he wants to write an article for the paper saying that math is the most popular subject at the school. Explain why this is not a valid conclusion and suggest a way to gather better data to determine what subject is most popular.

SBAC Sample Item:

Example Stem: David wants to estimate the number of students from his seventh grade class whose favorite subject is math. He needs to create a random sample of students. How should David collect his sample data?

A. David should ask 20 students in a math class.
B. David should ask 20 students on a school bus.
C. David should ask 20 students in seventh grade.
D. David should ask 20 students from the entire school.

7th Grade – CCSS for Mathematics

7.SP.A Use random sampling to draw inferences about a population.

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Students recognize that it is difficult to gather statistics on an entire population. They also learn that a random sample can be representative of the total population and will generate valid predictions. Students use this information to draw inferences from data (MP.1, MP.2, MP.3, MP.4, MP.5, MP.6, MP.7). The standards in the 7.SP.1–2 cluster represent opportunities to apply percentages and proportional reasoning. In order to make inferences about a population, one applies such reasoning to the sample and the entire population (CA *Mathematics Framework*, adopted Nov. 6, 2013).

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7.SP.A Use random sampling to draw inferences about a population.

7.SP.2 Use data from a random sample to draw inferences about population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Essential Skills and Concepts:
- ☐ Draw inferences about populations
- ☐ Create samples to predict variation

Question Stems and Prompts:
- ✓ What is the mean?
- ✓ What can you conclude from the results?
- ✓ What is the variation in the sample?

Math Vocabulary Tier 3	**Spanish Cognates**
• data	datos
• sample	muestra
• variation	variación

Standards Connections
7.SP.1 → *7.SP.2*
7.SP.2 → 7.SP.4

SBAC Sample Items:

Example Stem: A sandwich shop manager wants to estimate how many of each type of sandwich will be purchased in a month. The manager keeps track of the orders for one week. The table shows the results.

Type of Sandwich	Number Ordered
Roast Beef	152
Tuna	114
Turkey	209

Based on the data, select **all** the estimates that could represent the number of times each type of sandwich will be ordered in a month.

A. 608 roast beef, 456 tuna, 836 turkey
B. 857 roast beef, 467 tuna, 623 turkey
C. 654 roast beef, 490 tuna, 899 turkey
D. 479 roast beef, 878 tuna, 638 turkey
E. 1299 roast beef, 1003 tuna, 1782 turkey

Example Stem: A representative sample of 50 students from a high school is surveyed. Each student is asked what science class he or she is taking. The table shows the responses.

Science Class	Number of Students
Physics	6
Chemistry	10
Biology	18
Earth Science	4
Health Science	12

Select **all** the statements about the students at the high school that are valid based on the survey results.

A. Twice as many students are taking Health Science than are taking Physics.
B. 20% of students are taking Chemistry.
C. In a group of 25 students, it is expected that 4 of the students are taking Earth Science.
D. In a group of 150 students, it is expected that 18 of the students are taking Physics.

7th Grade – CCSS for Mathematics

7.SP.A Use random sampling to draw inferences about a population.

7.SP.2 Use data from a random sample to draw inferences about population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Essential Skills and Concepts:
- ☐ Draw inferences about populations
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Question Stems and Prompts:
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7.SP.A.2

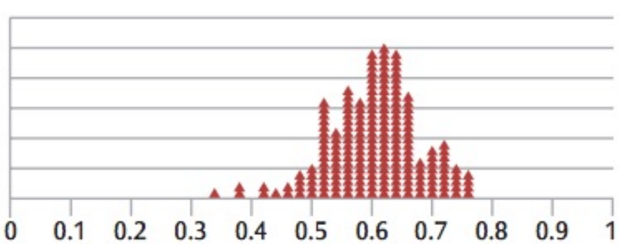
Standard Explanation

Students recognize that it is difficult to gather statistics on an entire population. They also learn that a random sample can be representative of the total population and will generate valid predictions. Students use this information to draw inferences from data (MP.1, MP.2, MP.3, MP.4, MP.5, MP.6, MP.7). The standards in the 7.SP.1–2 cluster represent opportunities to apply percentages and proportional reasoning. In order to make inferences about a population, one applies such reasoning to the sample and the entire population.

Variability in samples can be studied by using simulation (7.SP.2). Web-based software and spreadsheet programs may be used to run samples. For example, suppose students are using random sampling to determine the proportion of students who prefer football as their favorite sport, and suppose that 60% is the true proportion of the population. Students may simulate the sampling by conducting a simple experiment: place a collection of red and blue chips in a container in a ratio of 60:40, randomly select a chip 50 separate times with replacement, and record the proportion that came out red. If this experiment is repeated 200 times, students might obtain a distribution of the sample proportions similar to the one in figure 7-1.

This is a way for students to understand that the sample proportion can vary quite a bit, from as low as 45% to as high as 75%. Students can conjecture whether this variability will increase or decrease when the sample size increases, or if this variability depends on the true population proportion (MP.3) [adapted from UA Progressions Documents 2011e] (CA Mathematics Framework, adopted Nov. 6, 2013).

Figure 7-1. Results of Simulations



Proportions of red chips in 200 random samples of size 50 from a population in which 60% of the chips are red.

7th Grade – CCSS for Mathematics

7.SP.A.2

Standard Explanation

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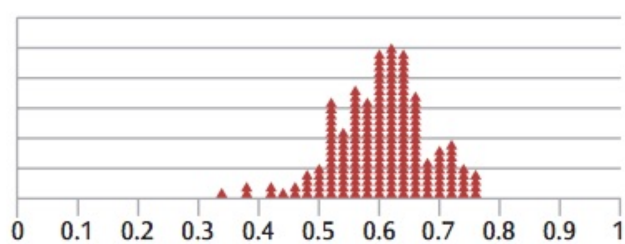

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7th Grade – CCSS for Mathematics

7.SP.B Draw informal comparative inferences about two populations.

7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

Essential Skills and Concepts:

- ☐ Make comparisons between two numerical data distributions
- ☐ Use measures of center and measures of variability to make statements that form the basis of informal comparative inferences

Question Stems and Prompts:

- ✓ How do the means of the two populations compare?
- ✓ Use the range to make comparisons about two data sets.
- ✓ How do the visual representations of the data sets compare?
- ✓ What inferences can you make about these two sample populations?

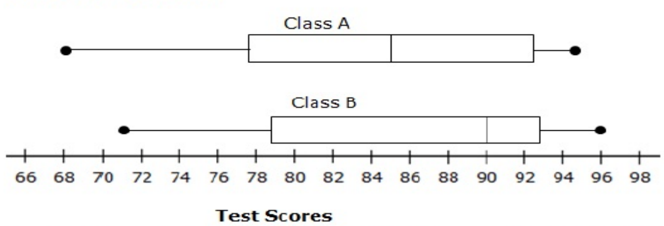
Vocabulary	**Spanish Cognates**
Tier 3	
• variability	variabilidad
• data	datos
• distributions	distribución

Standards Connections

5.NF.4, 6.NS.1, 6.SP.2 → *7.SP.3*
7.SP.3 → 7.SP.4

SBAC Sample Item:

Example Stem: The box plot shows a summary of test scores for Class A and Class B on the same exam. Both classes have the same number of students.



Determine whether each statement is true based on these box plots. Select True or False for each statement.

Statement	True	False
In each class, at least 25% of students scored below 80 on the test.		
The median test score of Class B is 5 points **less** than the median test score of Class A.		
In each class, more than 25% of students have test scores **greater** than 90.		

7.SP.B Draw informal comparative inferences about two populations.

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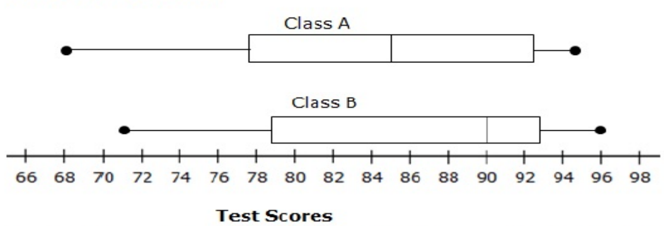

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7.SP.B Draw informal comparative inferences about two populations.

7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

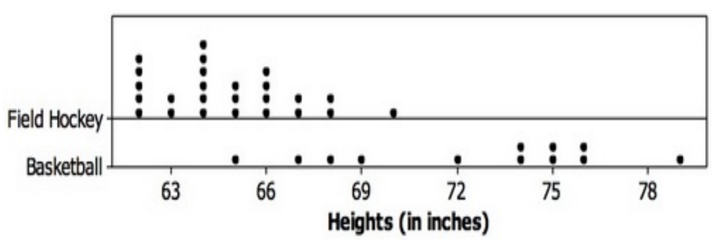
Standard Explanation

Comparing two data sets is a new concept for students (7.SP.3–4). Students build on their understanding of graphs, mean, median, mean absolute deviation (MAD), and interquartile range from sixth grade. They know that:

- understanding data requires consideration of the measures of variability as well as the mean or median;
- variability is responsible for the overlap of two data sets, and an increase in variability can increase the overlap;
- the median is paired with the interquartile range and the mean is paired with the mean absolute deviation (adapted from NCDPI 2013b) (CA Mathematics Framework, adopted Nov. 6, 2013).

7.SP.3-4 Illustrative Task:

- College Athletes https://www.illustrativemathematics.org/content-standards/7/SP/B/3/tasks/1340



a. Based on visual inspection of the dotplots, which group appears to have the larger average height? Which group appears to have the greater variability in the heights?

b. Compute the mean and mean absolute deviation (MAD) for each group. Do these values support your answers in part (a)?

7th Grade – CCSS for Mathematics

7.SP.B Draw informal comparative inferences about two populations.

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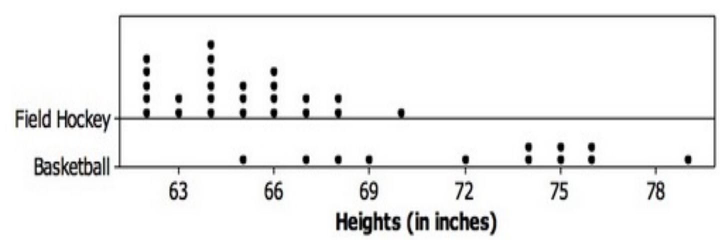

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7.SP.B Draw informal comparative inferences about two populations.

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Essential Skills and Concepts:

- ☐ Draw informal inferences about populations
- ☐ Use measures of center tendencies
- ☐ Use measures of variability

Question Stems and Prompts:

- ✓ Which measure of central tendency is best to use? How do you know?
- ✓ Compare the two populations using the median?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• variability	variabilidad
• median	mediana

Standards Connections

7.SP.2, 7.SP.3 → *7.SP.4*

7.SP.3-4 Examples:

Example: Comparing Two Populations 7.SP.3–4

College football teams are grouped with similar teams into divisions based on many factors. In terms of enrollment and revenue, schools from the Football Bowl Subdivision (FBS) are typically much larger than schools of other divisions. By contrast, Division III schools typically have smaller student populations and limited financial resources.

It is generally believed that, on average, the offensive linemen of FBS schools are heavier than those of Division III schools.

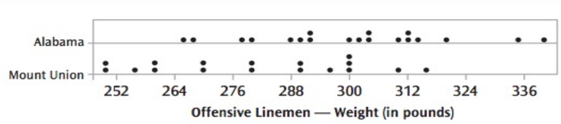
For the 2012 season, the University of Mount Union Purple Raiders football team won the Division III National Championship, and the University of Alabama Crimson Tide football team won the FBS National Championship. Following are the weights of the offensive linemen for both teams from that season.[8] A combined dot plot for both teams is also shown.

University of Alabama

277	265	292	303	303	320	300	313	267	288
311	280	302	335	310	290	312	340	292	

University of Mount Union

250	250	290	260	270	270	310	290	280	315
280	295	300	300	260	255	300			



Here are some examples of conclusions that may be drawn from the data and the dot plot:

a. Based on a visual inspection of the dot plot, the mean of the Alabama group seems to be higher than the mean of the Mount Union group. However, the overall spread of each distribution appears to be similar, so we might expect the *variability* to be similar as well.

b. The Alabama mean is 300 pounds, with a MAD of 15.68 pounds. The Mount Union mean is 280.88 pounds, with a MAD of 17.99 pounds.

7th Grade – CCSS for Mathematics

7.SP.B Draw informal comparative inferences about two populations.

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Essential Skills and Concepts:

- ☐ Draw informal inferences about populations
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- ☐ Use measures of variability

Question Stems and Prompts:

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Standards Connections

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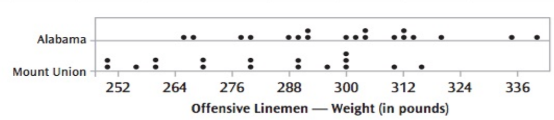
280	295	300	300	260	255	300			



Here are some examples of conclusions that may be drawn from the data and the dot plot:

a. Based on a visual inspection of the dot plot, the mean of the Alabama group seems to be higher than the mean of the Mount Union group. However, the overall spread of each distribution appears to be similar, so we might expect the *variability* to be similar as well.

b. The Alabama mean is 300 pounds, with a MAD of 15.68 pounds. The Mount Union mean is 280.88 pounds, with a MAD of 17.99 pounds.

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7.SP.B Draw informal comparative inferences about two populations.

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Standard Explanation
Comparing two data sets is a new concept for students (7.SP.3–4). Students build on their understanding of graphs, mean, median, mean absolute deviation (MAD), and interquartile range from sixth grade. They know that:

- understanding data requires consideration of the measures of variability as well as the mean or median;
- variability is responsible for the overlap of two data sets, and an increase in variability can increase the overlap;
- the median is paired with the interquartile range and the mean is paired with the mean absolute deviation (adapted from NCDPI 2013b) (CA Mathematics Framework, adopted Nov. 6, 2013).

7.SP.3-4 Illustrative Task:

- Offensive Lineman
  https://www.illustrativemathematics.org/content-standards/7/SP/B/3/tasks/1341

College football teams are grouped with similar teams into "divisions" (and in some cases, "subdivisions") based on many factors such as game attendance, level of competition, athletic department resources, and so on. Schools from the Football Bowl Subdivision (FBS, formerly known as Division 1-A) are typically much larger schools than schools of any other division in terms of enrollment and revenue. "Division III" is a division of schools with typically smaller enrollment and resources.

One particular position on a football team is called "offensive lineman," and it is generally believed that the offensive linemen of FBS schools are heavier on average than the offensive linemen of Division III schools.

For the 2012 season, the University of Mount Union Purple Raiders football team won the Division III National Football Championship while the University of Alabama Crimson Tide football team won the FBS National Championship. Below are the weights of the offensive linemen for both teams from that season.

(Accessed at http://athletics.mountunion.edu/sports/fball/2012-13/roster, http://www.rolltide.com/sports/m-footbl/mtt/alab-m-footbl-mtt.html on 1/14/13)

7th Grade – CCSS for Mathematics

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7.SP.C Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Essential Skills and Concepts:
- ☐ Understand probability
- ☐ Understand the chance of an event occurring

Question Stems and Prompts:
- ✓ What is the probability of an event occurring?
- ✓ What is the probability of an event not occurring?

Vocabulary	**Spanish Cognates**
Tier 3	
• probability	probabilidad
• event	evento
• likelihood	probabilidad

Standards Connections
7.SP.5 → 7.SP.1, 7.SP.6

7.SP.5 Example:

Focus, Coherence, and Rigor

Probability models draw on proportional reasoning and should be connected to major grade-seven work in the cluster "Analyze proportional relationships and use them to solve real-world and mathematical problems" (**7.RP.1–3▲**).

SBAC Sample Item:

Example Stem: A deck of 12 cards labeled 1 through 12 is shuffled. One card is selected at random.

Determine whether each statement correctly describes the likelihood of an event based on the given deck of cards. Select True or False for each statement.

Statement	True	False
It is impossible that a card with a number greater than 13 is selected.		
It is likely that a card with a number greater than 2 is selected.		
It is certain that a card with an odd or even number is selected.		
It is unlikely that a card with a number less than 7 is selected.		

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7th Grade – CCSS for Mathematics

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Standard Explanation

Seventh grade marks the first time students are formally introduced to probability. There are numerous modeling opportunities within this topic, and hands-on activities should predominate in the classroom. Technology can enhance the study of probability—for example, with online simulations of spinners, number cubes, and random number generators. The Internet is also a source of real data (e.g., on population, area, survey results, demographic information, and so forth) that can be used for writing and solving problems (CA Mathematics Framework, adopted Nov. 6, 2013).

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7.SP.C Investigate chance processes and develop, use, and evaluate probability models

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

Essential Skills and Concepts:
- ☐ Collect data to approximate the probability of an event
- ☐ Predict the frequency

Question Stems and Prompts:
- ✓ Predict the number of times an event will happen
- ✓ What is the long-term frequency of this event?

Vocabulary	**Spanish Cognates**
Tier 3	
• probability	probabilidad
• event	evento
• frequency	frecuencia

Standards Connections
7.RP.3, 7.SP.5 → *7.SP.6*
7.SP.6 → 7.SP.7

SBAC Sample Item:
Example Stem: This table shows outcomes of a spinner with 3 equal sections colored orange, blue, and white.

Section	Outcomes
Orange	30
Blue	34
White	36

Based on the outcomes, enter the number of times the arrow is expected to land on the orange section if it is spun 20 times.

7th Grade – CCSS for Mathematics

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7.SP.C.6

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7.SP.6 Illustrative Tasks:

- Heads or Tails
  https://www.illustrativemathematics.org/content-standards/7/SP/C/6/tasks/1521

Each of the 20 students in Mr. Anderson's class flipped a coin ten times and recorded how many times it came out heads.

a. How many heads do you think you will see out of ten tosses?

b. Would it surprise you to see 4 heads out of ten tosses? Explain why or why not.

c. Here are the results for the twenty students in Mr. Anderson's class. Use this data to estimate the probability of observing 4, 5 or 6 heads in ten tosses of the coin. (It might help to organize the data in a table or in a dot plot first.)

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number of heads	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

- Rolling Dice
  https://www.illustrativemathematics.org/content-standards/7/SP/C/6/tasks/1216

Roll two dice 10 times. After each roll, note whether any sixes were observed and record your results in the table below.

a. What fraction of the 10 rolls resulted in at least one six?

b. Combine your results with those of your classmates. What fraction of all the rolls in the class resulted in at least one six?

c. Make a list of all the different possible outcomes that might be observed when two dice are rolled. (Hint: There are 36 different possible outcomes.)

d. What fraction of the 36 possible outcomes result in at least one six?

e. Suppose you and your classmates were able to roll the two dice many thousands of times. What fraction of the time would you expect to roll at least one six?

7^{th} *Grade – CCSS for Mathematics*

7.SP.C.6

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7.SP.C Investigate chance processes and develop, use, and evaluate probability models.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

Essential Skills and Concepts:

- ☐ Calculate the probabilities of events
- ☐ Understand discrepancies found in events
- ☐ Analyze probabilities models

Question Stems and Prompts:

- ✓ What inferences could you conclude from the data collected?
- ✓ Explain the reasons for discrepancy regarding the probability?

Vocabulary	**Spanish Cognates**
Tier 2	
• observe	observar
• discrepancy	discrepancia
Tier 3	
• probability	probabilidad
• frequencies	frecuencia

Standards Connections

7.RP.3, 7.SP.6 → *7.SP.7*
7.SP.7 → 7.SP.8

7.SP.7 Example:

Example: A Simple Probability Model 7.SP.7

A box contains 10 red chips and 10 black chips. Without looking, each student reaches into the box and pulls out a chip. If each of the first 5 students pulls out (and keeps) a red chip, what is the probability that the sixth student will pull a red chip?

Solution: The events in question, pulling out a red or black chip, should be considered equally likely. Furthermore, though students new to probability may believe in the "gambler's fallacy"—that since 5 red chips have already been chosen, there is a very large chance that a black chip will be chosen next—students must still compute the probabilities of events as equally likely. There are 15 chips left in the box (5 red and 10 black), so the probability that the sixth student will select a red chip is $\frac{5}{15} = \frac{1}{3}$.

7th Grade – CCSS for Mathematics

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7.SP.C.7

Standard Explanation

Grade-seven students interpret probability as indicating the long-run relative frequency of the occurrence of an event. Students may use online simulations such as the following to support their understanding: • Marble Mania! (http://www.sciencenetlinks.com/interactives/marble/marblemania.html [Science NetLinks 2013]) • Random Drawing Tool (http://illuminations.nctm.org/activitydetail.aspx?id=67 [National Council of Teachers of Mathematics Illuminations 2013a]) Students develop and use probability models to find the probabilities of events and investigate both empirical probabilities (i.e., probabilities based on observing outcomes of a simulated random process) and theoretical probabilities (i.e., probabilities based on the structure of the sample space of an event) [7.SP.7] (CA Mathematics Framework, adopted Nov. 6, 2013).

7.SP.7 Illustrative Task:

- How many Buttons? https://www.illustrativemathematics.org/content-standards/7/SP/C/7/tasks/1022

Look at the shirt you are wearing today, and determine how many buttons it has. Then complete the following table for all the members of your class.

	No Buttons	One or Two Buttons	Three or Four Buttons	More Than Four Buttons
Male				
Female				

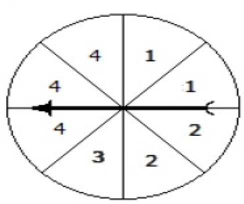
Suppose each student writes his or her name on an index card, and one card is selected randomly.

a. What is the probability that the student whose card is selected is wearing a shirt with no buttons?

b. What is the probability that the student whose card is selected is female and is wearing a shirt with two or fewer buttons?

SBAC Sample Items:

Example Stem: This spinner is divided into 8 equal-sized sections.



Enter the probability of the arrow landing on a section labeled 2 on the first spin.

Example Stem: This table shows the results of randomly selecting colored marbles from a bag 20 times.

	Red	Yellow	Blue	Orange	Purple	Green
Number of Times Selected	7	4	3	1	0	5

Based on these results, enter the expected probability of selecting a red marble from the bag in one attempt.

7th Grade – CCSS for Mathematics

7.SP.C.7

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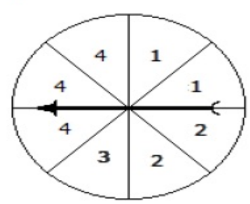

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7th Grade – CCSS for Mathematics

7.SP.C Investigate chance processes and develop, use, and evaluate probability models

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Essential Skills and Concepts:

- ☐ Understand compound events
- ☐ Identify outcomes of an event
- ☐ Represent outcomes
- ☐ Design compound events

Question Stems and Prompts:

- ✓ What is the difference between a compound and simple event? What is the difference in finding outcomes for both events?
- ✓ Explain what the probability represents?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• simple event	eventos simples
• compound event	eventos compuestos
• frequencies	frecuencias
• outcomes	

Standards Connections

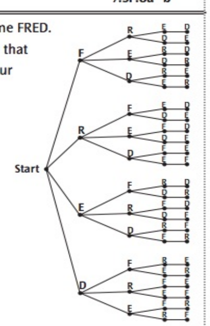
7.RP.3, 7.SP.7 → *7.SP.8*

7.SP.8 Examples:

Example: Tree Diagrams — **7.SP.8a–b**

Using a tree diagram, show all possible arrangements of the letters in the name FRED. If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order? What is the probability that your "word" will have an F as the first letter?

Solution: A tree diagram reveals that, out of 24 total outcomes, there is only one outcome where the letters F-R-E-D appear in that order, so the probability of the event occurring is $\frac{1}{24}$. Regarding the second question, the entire top branch (6 outcomes) represents the outcomes where the first letter is F, so the probability of that occurring is $\frac{6}{24} = \frac{1}{4}$.



Adapted from ADE 2010.

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Essential Skills and Concepts:

- ☐ Understand compound events
- ☐ Identify outcomes of an event
- ☐ Represent outcomes
- ☐ Design compound events

Question Stems and Prompts:

- ✓ What is the difference between a compound and simple event? What is the difference in finding outcomes for both events?
- ✓ Explain what the probability represents?

Math Vocabulary	**Spanish Cognates**
Tier 3	
• simple event	eventos simples
• compound event	eventos compuestos
• frequencies	frecuencias
• outcomes	

Standards Connections

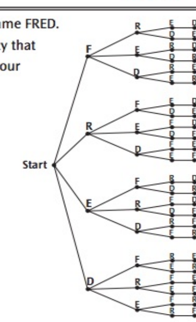
7.RP.3, 7.SP.7 → *7.SP.8*

7.SP.8 Examples:

Example: Tree Diagrams — **7.SP.8a–b**

Using a tree diagram, show all possible arrangements of the letters in the name FRED. If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order? What is the probability that your "word" will have an F as the first letter?

Solution: A tree diagram reveals that, out of 24 total outcomes, there is only one outcome where the letters F-R-E-D appear in that order, so the probability of the event occurring is $\frac{1}{24}$. Regarding the second question, the entire top branch (6 outcomes) represents the outcomes where the first letter is F, so the probability of that occurring is $\frac{6}{24} = \frac{1}{4}$.



Adapted from ADE 2010.

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7.SP.C.8

Standard Explanation

Students in grade seven also examine compound events (such as tossing a coin and rolling a standard number cube) and use basic counting ideas for finding the total number of equally likely outcomes for such an event. For example, 2 outcomes for the coin and 6 outcomes for the number cube result in 12 total outcomes. At this grade level, there is no need to introduce formal methods of finding permutations and combinations. Students also use various means of organizing the outcomes of an event, such as two-way tables or tree diagrams (7.SP.8a–b).

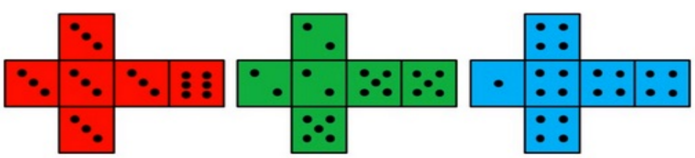
Finally, students in grade seven use simulations to determine probabilities (frequencies) for compound events (7.SP.8c). For a more complete discussion of the Statistics and Probability domain, see “Progressions Documents for the Common Core Math Standards: Draft 6–8 Progression on Statistics and Probability” (http://ime.math.arizona.edu/progressions/ [UA Progressions Documents 2011e]) (CA Mathematics Framework, adopted Nov. 6, 2013).

7.SP.8 Illustrative Task:

- Red, Green, or Blue? https://www.illustrativemathematics.org/content-standards/7/SP/C/8/tasks/1442

You have three dice; one is red, one is green, and one is blue. These dice are different than regular six-sided dice, which show each of the numbers 1 to 6 exactly once. The red die, for example, has 3 dots on each of five sides, and 6 dots on the other. The number of dots on each side are shown in the table and picture below.

Red	3	3	3	3	3	6
Green	2	2	2	5	5	5
Blue	1	4	4	4	4	4



To play the game, each person picks one of the three dice. However, they have to pick different colors.

- The two players both roll their dice. The highest number wins the round.
- The players roll their dice 30 times, keeping track of who wins each round.
- Whoever has won the greatest number of rounds after 30 rolls wins the game.

a. Who is more likely to win when a person with the red die plays against a person with the green die? What about green vs. blue? What about blue vs. red?

b. Would you rather be the first person to pick a die or the second person? Explain.

7th Grade – CCSS for Mathematics

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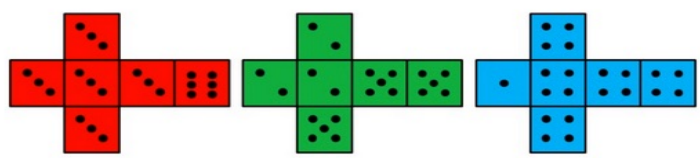

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Resources for the CCSS 7th Grade Bookmarks

California *Mathematics Framework,* adopted by the California State Board of Education November 6, 2013, http://www.cde.ca.gov/ci/ma/cf/draft2mathfwchapters.asp

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

Common Core Standards Writing Team. Progressions for the Common Core State Standards in Mathematics Tucson, AZ: Institute for Mathematics and Education, University of Arizona (Drafts)

- The Number System, 6 – 8 (2013, July 9)
- 6 – 8, Expressions and Equations (2011, April 22)
- Grade 8, High School, Functions, (2012, December 3)
- 6 – 8, Statistics and Probability (2011, December 26)
- 6 – 7, Ratios and Proportional Relationships (2011, December 26)

Illustrative Mathematics™ was originally developed at the University of Arizona (2011), nonprofit corporation (2013), Illustrative Tasks, http://www.illustrativemathematics.org/

Student Achievement Partners, Achieve the Core http://achievethecore.org/, Focus by Grade Level, http://achievethecore.org/dashboard/300/search/1/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/774/focus-by-grade-level

North Carolina Department of Public Instruction, Instructional Support Tools for Achieving New Standards, Math Unpacking Standards 2012, http://www.ncpublicschools.org/acre/standards/common-core-tools/ - unmath

Common Core Flipbooks 2012, Kansas Association of Teachers of Mathematics (KATM) http://www.katm.org/baker/pages/common-core-resources.php

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Arizona's College and Career Ready Standards – Mathematics – Kindergarten, Arizona Department of Education – High Academic Standards for Students Arizona's College and Career Ready Standards – Mathematics, State Board Approved June 2010 October 2013 Publication, http://www.azed.gov/azccrs/mathstandards/

Howard County Public School System, Elementary Mathematics Office, Standards for Mathematical Practice for Parents, Draft 2011, https://grade3commoncoremath.wikispaces.hcpss.org/file/view/SFMP for Parents.docx/286906254/SFMP for Parents.docx

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Long Beach Unified School District, Math Cognates, retrieved on 7/14/14, http://www.lbschools.net/Main_Offices/Curriculum/Areas/Mathematics/XCD/ListOfMathCognates.pdf

A Graph of the Content Standards, Jason Zimba, June 7, 2012, http://tinyurl.com/ccssmgraph

Smarter Balanced Mathematics Item Specifications, retrieved 6/19/15, http://www.smarterbalanced.org/smarter-balanced-assessments/

Arizona's College and Career Ready Standards – Mathematics – Kindergarten, Arizona Department of Education – High Academic Standards for Students Arizona's College and Career Ready Standards – Mathematics, State Board Approved June 2010 October 2013 Publication, http://www.azed.gov/azccrs/mathstandards/

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