Supporting California Standards
Mathematics Continuum: Grades 2-4

Tulare County Office of Education
Tim A. Hire, County Superintendent of Schools

Operations and Algebraic Thinking

Grade 2

Represent and solve problems involving addition and subtraction.

2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] {[1] See Glossary, Table 1.}

Add and subtract within 20.

2.OA.2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers. {[2] See standard 1.OA.6 for a list of mental strategies.}

Work with equal groups of objects to gain foundations for multiplication.

2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Grade 3

Represent and solve problems involving multiplication and division.

3.OA.1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5 × 7.

3.OA.2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56÷8.*

3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1] [[1] See Glossary, Table 2.]

3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* [[2]Students need not use formal terms for these properties.]

3.OA.6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

3.OA.7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3] [[3]This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).]

3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Grade 4

Use the four operations with whole numbers to solve problems.

4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] [[1]See Glossary, Table 2]

4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Created by Educational Resource Services, Tulare County Office of Education, Visalia, California (559) 651-3031

Number and Operations in Base Ten

Grade 2	Grade 3	Grade 4 Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
Understand place value. 2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). 2.NBT.2. Count within 1000; skip-count by 2s, 5s, 10s, and 100s. CA 2.NBT.3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. 2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. **Use place value understanding and properties of operations to add and subtract.** 2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. 2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations. 2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **2.NBT.7.1 Use estimation strategies to make reasonable estimates in problem solving. CA** 2.NBT.8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. 2.NBT.9. Explain why addition and subtraction strategies work, using place value and the properties of operations.3 {3 Explanations may be supported by drawings of objects.}	**Use place value understanding and properties of operations to perform multi-digit arithmetic.[4] [4A range of algorithms may be used]** 3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100. 3.NBT.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**Generalize place value understanding for multi-digit whole numbers.** 4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place. **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 4.NBT.4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. 4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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Number and Operations - Fractions		
Grade 2	**Grade 3** Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.	**Grade 4** Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
None	**Develop understanding of fractions as numbers.** 3.NF.1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. 3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. 3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**Extend understanding of fraction equivalence and ordering.** 4.NF.1. Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** 4.NF.3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. 4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **Understand decimal notation for fractions, and compare decimal fractions.** 4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* [[4]Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.] 4.NF.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* 4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using **the number line or another** visual model. **CA**

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Measurement and Data

Grade 2

Measure and estimate lengths in standard units.

2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

2.MD.3. Estimate lengths using units of inches, feet, centimeters, and meters.

2.MD.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

2.MD.5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **Know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year). CA**

2.MD.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

2.MD.9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph. {[4] See Glossary, Table 1.}

Grade 3

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7] [[6] Excludes compound units such as cm^3 and finding the geometric volume of a container. [7]Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

Represent and interpret data.

3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7. Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Grade 4

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4.MD.4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Measurement and Data		
Grade 2	**Grade 3**	**Grade 4**
	b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** 3.MD.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	

Geometry		
Grade 2	**Grade 3**	**Grade 4**
Reason with shapes and their attributes. 2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. {[5] Sizes are compared directly or visually, not compared by measuring.} 2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.**	**Reason with shapes and their attributes.** 3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. 3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **(Two dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA** 4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Standards for Mathematical Practice (K-12)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.