

Operations and Algebraic Thinking

Grade 3	Grade 4	Grade 5
<p><b>Represent and solve problems involving multiplication and division.</b></p> <p>3.OA.1. Interpret products of whole numbers, e.g., interpret <math>5 \times 7</math> as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as <math>5 \times 7</math>.</i></p> <p>3.OA.2. Interpret whole-number quotients of whole numbers, e.g., interpret <math>56 \div 8</math> as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. <i>For example, describe a context in which a number of shares or a number of groups can be expressed as <math>56 \div 8</math>.</i></p> <p>3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.<sup>1</sup> [See Glossary, Table 2.]</p> <p>3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations <math>8 \times ? = 48</math>, <math>5 = \square \div 3</math>, <math>6 \times 6 = ?</math>.</p> <p><b>Understand properties of multiplication and the relationship between multiplication and division.</b></p> <p>3.OA.5. Apply properties of operations as strategies to multiply and divide.<sup>2</sup> <i>Examples: If <math>6 \times 4 = 24</math> is known, then <math>4 \times 6 = 24</math> is also known. (Commutative property of multiplication.) <math>3 \times 5 \times 2</math> can be found by <math>3 \times 5 = 15</math>, then <math>15 \times 2 = 30</math>, or by <math>5 \times 2 = 10</math>, then <math>3 \times 10 = 30</math>. (Associative property of multiplication.) Knowing that <math>8 \times 5 = 40</math> and <math>8 \times 2 = 16</math>, one can find <math>8 \times 7</math> as <math>8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56</math>. (Distributive property.)</i> [<sup>2</sup>Students need not use formal terms for these properties.]</p> <p>3.OA.6. Understand division as an unknown-factor problem. <i>For example, find <math>32 \div 8</math> by finding the number that makes 32 when multiplied by 8.</i></p> <p><b>Multiply and divide within 100.</b></p> <p>3.OA.7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that <math>8 \times 5 = 40</math>, one knows <math>40 \div 5 = 8</math>) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.</p> <p><b>Solve problems involving the four operations, and identify and explain patterns in arithmetic.</b></p> <p>3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.<sup>3</sup> [<sup>3</sup>This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).]</p> <p>3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i></p>	<p><b>Use the four operations with whole numbers to solve problems.</b></p> <p>4.OA.1. Interpret a multiplication equation as a comparison, e.g., interpret <math>35 = 5 \times 7</math> as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</p> <p>4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.<sup>1</sup> [See Glossary, Table 2]</p> <p>4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p> <p><b>Gain familiarity with factors and multiples.</b></p> <p>4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p> <p><b>Generate and analyze patterns.</b></p> <p>4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i></p>	<p><b>Write and interpret numerical expressions.</b></p> <p>5.OA.1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</p> <p>5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as <math>2 \times (8 + 7)</math>. Recognize that <math>3 \times (18932 + 921)</math> is three times as large as <math>18932 + 921</math>, without having to calculate the indicated sum or product.</i></p> <p><b>5.OA.2.1 Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as <math>2 \times 2 \times 2 \times 3</math>. CA</b></p> <p><b>Analyze patterns and relationships.</b></p> <p>5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. <i>For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</i></p>

Number and Operations in Base Ten		
Grade 3	Grade 4	Grade 5
<p><b>Use place value understanding and properties of operations to perform multi-digit arithmetic.</b><sup>4</sup> [4A range of algorithms may be used]</p> <p>3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100.</p> <p>3.NBT.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p><b>3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., <math>9 \times 80</math>, <math>5 \times 60</math>) using strategies based on place value and properties of operations.</b></p>	<p>Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.</p> <p><b>Generalize place value understanding for multi-digit whole numbers.</b></p> <p>4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <i>For example, recognize that <math>700 \div 70 = 10</math> by applying concepts of place value and division.</i></p> <p>4.NBT.2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p> <p>4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place.</p> <p><b>Use place value understanding and properties of operations to perform multi-digit arithmetic.</b></p> <p>4.NBT.4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p> <p>4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p><b>Understand the place value system.</b></p> <p>5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and <math>1/10</math> of what it represents in the place to its left.</p> <p>5. NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p> <p>5. NBT.3. Read, write, and compare decimals to thousandths.</p> <p>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., <math>347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)</math>.</p> <p>b. Compare two decimals to thousandths based on meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p> <p>5. NBT.4. Use place value understanding to round decimals to any place.</p> <p><b>Perform operations with multi-digit whole numbers and with decimals to hundredths.</b></p> <p>5.NBT.5. Fluently multiply multi-digit whole numbers using the standard algorithm.</p> <p>5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>

Number and Operations - Fractions		
Grade 3	Grade 4	Grade 5
Limited to fractions with denominators 2, 3, 4, 6, and 8.	Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	
<p><b>Develop understanding of fractions as numbers.</b></p> <p>3.NF.1. Understand a fraction <math>1/b</math> as the quantity formed by 1 part when a whole is partitioned into <math>b</math> equal parts; understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</p> <p>3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. Represent a fraction <math>1/b</math> on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into <math>b</math> equal parts. Recognize that each part has size <math>1/b</math> and that the endpoint of the part based at 0 locates the number <math>1/b</math> on the number line.</p> <p>b. Represent a fraction <math>a/b</math> on a number line diagram by marking off <math>a</math> lengths <math>1/b</math> from 0. Recognize that the resulting interval has size <math>a/b</math> and that its endpoint locates the number <math>a/b</math> on the number line.</p> <p>3.NF.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> <p>b. Recognize and generate simple equivalent fractions, e.g., <math>1/2 = 2/4</math>, <math>4/6 = 2/3</math>. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form <math>3 = 3/1</math>; recognize that <math>6/1 = 6</math>; locate <math>4/4</math> and 1 at the same point of a number line diagram.</p> <p>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model.</p>	<p><b>Extend understanding of fraction equivalence and ordering.</b></p> <p>4.NF.1. Explain why a fraction <math>a/b</math> is equivalent to a fraction <math>(n \times a)/(n \times b)</math> by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p> <p>4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as <math>1/2</math>. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model.</p> <p><b>Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</b></p> <p>4.NF.3. Understand a fraction <math>a/b</math> with <math>a &gt; 1</math> as a sum of fractions <math>1/b</math>.</p> <p>a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p> <p>b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: <math>3/8 = 1/8 + 1/8 + 1/8</math>; <math>3/8 = 1/8 + 2/8</math>; <math>2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8</math>.</p> <p>c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p> <p>d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</p> <p>4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p> <p>a. Understand a fraction <math>a/b</math> as a multiple of <math>1/b</math>. For example, use a visual fraction model to represent <math>5/4</math> as the product <math>5 \times (1/4)</math>, recording the conclusion by the equation <math>5/4 = 5 \times (1/4)</math>.</p> <p>b. Understand a multiple of <math>a/b</math> as a multiple of <math>1/b</math>, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express <math>3 \times (2/5)</math> as <math>6 \times (1/5)</math>, recognizing this product as <math>6/5</math>. (In general, <math>n \times (a/b) = (n \times a)/b</math>.)</p> <p>c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat <math>3/8</math> of a pound</p>	<p><b>Use equivalent fractions as a strategy to add and subtract fractions.</b></p> <p>5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, <math>2/3 + 5/4 = 8/12 + 15/12 = 23/12</math>. (In general, <math>a/b + c/d = (ad + bc)/bd</math>.)</p> <p>5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result <math>2/5 + 1/2 = 3/7</math>, by observing that <math>3/7 &lt; 1/2</math>.</p> <p><b>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</b></p> <p>5.NF.3. Interpret a fraction as division of the numerator by the denominator (<math>a/b = a \div b</math>). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret <math>3/4</math> as the result of dividing 3 by 4, noting that <math>3/4</math> multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size <math>3/4</math>. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</p> <p>5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p>a. Interpret the product <math>(a/b) \times q</math> as a parts of a partition of <math>q</math> into <math>b</math> equal parts; equivalently, as the result of a sequence of operations <math>a \times q \div b</math>. For example, use a visual fraction model to show <math>(2/3) \times 4 = 8/3</math>, and create a story context for this equation. Do the same with <math>(2/3) \times (4/5) = 8/15</math>. (In general, <math>(a/b) \times (c/d) = ac/bd</math>.)</p> <p>b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</p> <p>5.NF.5. Interpret multiplication as scaling (resizing), by:</p> <p>a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</p> <p>b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence <math>a/b = (n \times a)/(n \times b)</math> to the effect of multiplying <math>a/b</math> by 1.</p>

Number and Operations – Fractions (continued)		
Grade 3	Grade 4	Grade 5
Limited to fractions with denominators 2, 3, 4, 6, and 8.	Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.	
	<p><i>of roast beef will be needed? Between what two whole numbers does your answer lie?</i></p> <p><b>Understand decimal notation for fractions, and compare decimal fractions.</b></p> <p>4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.<sup>4</sup> For example, express <math>\frac{3}{10}</math> as <math>\frac{30}{100}</math>, and add <math>\frac{3}{10} + \frac{4}{100} = \frac{34}{100}</math>. [<sup>4</sup>Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.]</p> <p>4.NF.6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite <math>0.62</math> as <math>\frac{62}{100}</math>; describe a length as <math>0.62</math> meters; locate <math>0.62</math> on a number line diagram.</p> <p>4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using <b>the number line or another</b> visual model. <b>CA</b></p>	<p>5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</p> <p>5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.<sup>1</sup> [<sup>1</sup>Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.]</p> <p>a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for <math>(\frac{1}{3}) \div 4</math>, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that <math>(\frac{1}{3}) \div 4 = \frac{1}{12}</math> because <math>(\frac{1}{12}) \times 4 = \frac{1}{3}</math>.</p> <p>b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for <math>4 \div (\frac{1}{5})</math>, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that <math>4 \div (\frac{1}{5}) = 20</math> because <math>20 \times (\frac{1}{5}) = 4</math>.</p> <p>c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share <math>\frac{1}{2}</math> lb of chocolate equally? How many <math>\frac{1}{3}</math>-cup servings are in 2 cups of raisins?</p>

Measurement and Data		
Grade 3	Grade 4	Grade 5
<p><b>Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</b></p> <p>3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p> <p>3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).<sup>6</sup> Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.<sup>7</sup> [<sup>6</sup> Excludes compound units such as <math>\text{cm}^3</math> and finding the geometric volume of a container. <sup>7</sup>Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).</p> <p><b>Represent and interpret data.</b></p> <p>3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. <i>For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i></p> <p>3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.</p> <p><b>Geometric measurement: understand concepts of area and relate area to multiplication and to addition.</b></p> <p>3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <ol style="list-style-type: none"> <li>A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</li> <li>A plane figure which can be covered without gaps or overlaps by <math>n</math> unit squares is said to have an area of <math>n</math> square units.</li> </ol> <p>3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).</p> <p>3.MD.7 Relate area to the operations of multiplication and addition.</p> <ol style="list-style-type: none"> <li>Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.</li> </ol>	<p><b>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</b></p> <p>4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p> <p>4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p> <p>4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</i></p> <p><b>Represent and interpret data.</b></p> <p>4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p> <p><b>Geometric measurement: understand concepts of angle and measure angles.</b></p> <p>4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p> <ol style="list-style-type: none"> <li>An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through <math>\frac{1}{360}</math> of a circle is called a “one-degree angle,” and can be used to measure angles.</li> <li>An angle that turns through <math>n</math> one-degree angles is said to have an angle measure of <math>n</math> degrees.</li> </ol> <p>4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p>	<p><b>Convert like measurement units within a given measurement system.</b></p> <p>5.MD.1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p> <p><b>Represent and interpret data.</b></p> <p>5.MD.2. Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>). Use operations on fractions for this grade to solve problems involving information presented in line plots. <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i></p> <p><b>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</b></p> <p>5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</p> <ol style="list-style-type: none"> <li>A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.</li> <li>A solid figure which can be packed without gaps or overlaps using <math>n</math> unit cubes is said to have a volume of <math>n</math> cubic units.</li> </ol> <p>5.MD.4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.</p> <p>5.MD.5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</p> <ol style="list-style-type: none"> <li>Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</li> <li>Apply the formulas <math>V = l \times w \times h</math> and <math>V = b \times h</math> for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</li> <li>Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</li> </ol>

Supporting California Standards  
Mathematics Continuum: Grades 3-5

Measurement and Data		
Grade 3	Grade 4	Grade 5
<p>b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.</p> <p>c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths <math>a</math> and <math>b + c</math> is the sum of <math>a \times b</math> and <math>a \times c</math>. Use area models to represent the distributive property in mathematical reasoning.</p> <p>d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</p> <p><b>Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.</b></p> <p>3.MD.8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</p>	<p>4.MD.7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p>	

Geometry		
Grade 3	Grade 4	Grade 5
<p><b>Reason with shapes and their attributes.</b></p> <p>3.G.1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p> <p>3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. <i>For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</i></p>	<p><b>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</b></p> <p>4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p> <p>4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. <b>(Two dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA</b></p> <p>4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>	<p><b>Graph points on the coordinate plane to solve real-world and mathematical problems.</b></p> <p>5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</p> <p>5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</p> <p><b>Classify two-dimensional figures into categories based on their properties.</b></p> <p>5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</i></p> <p>5.G.4. Classify two-dimensional figures in a hierarchy based on properties.</p>

### Standards for Mathematical Practice (K-12)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.