Standards for Mathematical Practice (K-12)

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others
- 4. Model with mathematics

- 5. Use appropriate tools strategically
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

Ratios and Proportional Relationships		
Grade 6	Grade 7	Grade 8
 Understand ratio concepts and use ratio reasoning to solve problems. 6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." 6.RP.2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."¹¹ [1 Expectations for unit rates in this grade are limited to noncomplex fractions.] 6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. 	 Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ^{1/2}/_{1/4} miles per hour, equivalently 2 miles per hour. 7.RP.2. Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. 7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. 	See 8.EE.5

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The Number System		
Grade 6	Grade 7	Grade 8
 Grade 6 Apply and extend previous understandings of multiplication and division to divide fractions by fractions. 6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? Compute fluently with multi-digit numbers and find common factors and multiples. 6.NS.2. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. 6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. 6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers to the system of rational numbers. 6.NS.5. Understand that positive and negative numbers to the system of rational numbers. 6.NS.6. Understand that positive and negative numbers in a eveloge duantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below zero verses, explaining the meaning of 0 in each situation. 6.NS.6. Understand a rational numbers a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent poi		Grade 8 Know that there are numbers that are not rational, and approximate them by rational numbers. 8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. 8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers,
	a. Understand that multiplication is extended from fractions to rational	compare the size of

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Expressions and Equations		
Grade 6	Grade 7	Grade 8
Apply and extend previous understandings of arithmetic to algebraic expressions.	Use properties of operations to generate equivalent expressions.	Work with radicals and integer exponents.
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.	7.EE.1. Apply properties of operations as strategies to add, subtract,	8.EE.1. Know and apply the properties of integer exponents to generate
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.	factor, and expand linear expressions with rational coefficients.	equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/3^3$
 Write expressions that record operations with numbers and with letters 	7.EE.2. Understand that rewriting an expression in different forms in a	1/27.
standing for numbers. For example, express the calculation "Subtract y from	problem context can shed light on the problem and how the	8.EE.2. Use square root and cube root symbols to represent solutions to
5" as 5 – <i>y.</i>	quantities in it are related. For example, a + 0.05a = 1.05a means	equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational
b. Identify parts of an expression using mathematical terms (sum, term, product,	that "increase by 5%" is the same as "multiply by 1.05."	number. Evaluate square roots of small perfect squares and cube roots
factor, quotient, coefficient); view one or more parts of an expression as a	Solve real-life and mathematical problems using numerical and	of small perfect cubes. Know that $\sqrt{2}$ is irrational.
single entity. For example, describe the expression 2 (8 + 7) as a product of	algebraic expressions and equations.	8.EE.3. Use numbers expressed in the form of a single digit times an integer
two factors; view (8 + 7) as both a single entity and a sum of two terms.	7.EE.3. Solve multi-step real-life and mathematical problems posed with	power of 10 to estimate very large or very small quantities, and to
c. Evaluate expressions at specific values of their variables. Include expressions	positive and negative rational numbers in any form (whole	express how many times as much one is than the other. For example,
that arise from formulas used in real-world problems. Perform arithmetic	numbers, fractions, and decimals), using tools strategically. Apply	estimate the population of the United States as 3×10^8 and the
operations, including those involving whole-number exponents, in the	properties of operations to calculate with numbers in any form;	population of the world as 7×10^9 , and determine that the world
conventional order when there are no parentheses to specify a particular	convert between forms as appropriate; and assess the	population is more than 20 times larger.
order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6$	reasonableness of answers using mental computation and	8.EE.4. Perform operations with numbers expressed in scientific notation,
s^2 to find the volume and surface area of a cube with sides of length $s = 1/2$.	estimation strategies. For example: If a woman making \$25 an	including problems where both decimal and scientific notation are used.
6.EE.3. Apply the properties of operations to generate equivalent expressions. For	hour gets a 10% raise, she will make an additional 1/10 of her	Use scientific notation and choose units of appropriate size for
example, apply the distributive property to the expression $3(2 + x)$ to produce the	salary an hour, or \$2.50, for a new salary of \$27.50. If you want to	measurements of very large or very small quantities (e.g., use millimeters
equivalent expression 6 + 3x; apply the distributive property to the expression 24x	place a towel bar 9 3/4 inches long in the center of a door that is	per year for seafloor spreading). Interpret scientific notation that has been
+ 18y to produce the equivalent expression 6 $(4x + 3y)$; apply properties of	27 1/2 inches wide, you will need to place the bar about 9 inches	generated by technology.
operations to $y + y + y$ to produce the equivalent expression 3y.	from each edge; this estimate can be used as a check on the exact	Understand the connections between proportional relationships, lines, and
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions	computation.	linear equations.
name the same number regardless of which value is substituted into them). For	7.EE.4. Use variables to represent quantities in a real-world or	8.EE.5. Graph proportional relationships, interpreting the unit rate as the slope of
example, the expressions $y + y + y$ and $3y$ are equivalent because they name the	mathematical problem, and construct simple equations and	the graph. Compare two different proportional relationships represented
same number regardless of which number y stands for.	inequalities to solve problems by reasoning about the quantities.	in different ways. For example, compare a distance-time graph to a
Reason about and solve one-variable equations and inequalities.	a. Solve word problems leading to equations of the form $px + q =$	distance-time equation to determine which of two moving objects has
6.EE.5. Understand solving an equation or inequality as a process of answering a	r and $p(x + q) = r$, where p, q, and r are specific rational	greater speed.
question: which values from a specified set, if any, make the equation or inequality	numbers. Solve equations of these forms fluently. Compare an	8.EE.6. Use similar triangles to explain why the slope <i>m</i> is the same between any
true? Use substitution to determine whether a given number in a specified set	algebraic solution to an arithmetic solution, identifying the	two distinct points on a non-vertical line in the coordinate plane; derive
makes an equation or inequality true.	sequence of the operations used in each approach. For	the equation $y = mx$ for a line through the origin and the equation $y = mx$
6.EE.6. Use variables to represent numbers and write expressions when solving a real-	example, the perimeter of a rectangle is 54 cm. Its length is 6	+ <i>b</i> for a line intercepting the vertical axis at <i>b</i> .
world or mathematical problem; understand that a variable can represent an	cm. What is its width?	Analyze and solve linear equations and pairs of simultaneous linear
unknown number, or, depending on the purpose at hand, any number in a	b. Solve word problems leading to inequalities of the form px + q	equations. 8.EE.7. Solve linear equations in one variable.
specified set. 6.EE.7. Solve real-world and mathematical problems by writing and solving equations of	> r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret	a. Give examples of linear equations in one variable with one solution,
the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative	it in the context of the problem. For example: As a salesperson,	infinitely many solutions, or no solutions. Show which of these
rational numbers.	you are paid \$50 per week plus \$3 per sale. This week you	possibilities is the case by successively transforming the given
6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition	want your pay to be at least \$100. Write an inequality for the	equation into simpler forms, until an equivalent equation of the form $x =$
in a real-world or mathematical problem. Recognize that inequalities of the form x	number of sales you need to make, and describe the solutions.	a, a = a, or a = b results (where a and b are different numbers).
	number of sales you need to make, and describe the solutions.	a, a - a, or a - b results (where a and b are unificial numbers).

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 <i>c</i> or <i>x</i> < <i>c</i> have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Represent and analyze quantitative relationships between dependent and independent variables. 6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable. Analyze the relationship between the dependent and independent variable. Analyze the relationship between the dependent and independent variable using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.</i> 	 b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. 8.EE.8. Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations. Intersection setting the equations in two variables correspond to points of intersection statisfy both equations. b. Solve systems of two linear equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

	Functions		
Grade 6	Grade 7	Grade 8	
None	None	 Define, evaluate, and compare functions. 8.F.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. [Function notation is not required in Grade 8.] 8.F.2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. 8.F.3. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s² giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line. Use functions to model relationships between quantities. 8.F.4. Construct a function from a description of a relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. 	

Geometry		
Grade 6	Grade 7	Grade 8
Solve real-world and mathematical problems	Draw, construct, and describe geometrical figures and	Understand congruence and similarity using physical models, transparencies, or geometry
involving area, surface area, and volume.	describe the relationships between them.	software.
6.G.1. Find the area of right triangles, other triangles,	7.G.1. Solve problems involving scale drawings of geometric	8.G.1. Verify experimentally the properties of rotations, reflections, and translations:
special quadrilaterals, and polygons by composing	figures, including computing actual lengths and areas	a. Lines are taken to lines, and line segments to line segments of the same length.
into rectangles or decomposing into triangles and	from a scale drawing and reproducing a scale drawing	b. Angles are taken to angles of the same measure.
other shapes; apply these techniques in the	at a different scale.	c. Parallel lines are taken to parallel lines.
context of solving real-world and mathematical	7.G.2. Draw (freehand, with ruler and protractor, and with	8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained
problems.	technology) geometric shapes with given conditions.	from the first by a sequence of rotations, reflections, and translations; given two congruent figures,
6.G.2. Find the volume of a right rectangular prism with	Focus on constructing triangles from three measures	describe a sequence that exhibits the congruence between them.
fractional edge lengths by packing it with unit	of angles or sides, noticing when the conditions	8.G.3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures
cubes of the appropriate unit fraction edge lengths,	determine a unique triangle, more than one triangle, or	using coordinates.
and show that the volume is the same as would be	no triangle. 7.G.3. Describe the two-dimensional figures that result from	8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from
found by multiplying the edge lengths of the prism.	7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections	the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them.
Apply the formulas $V = I w h$ and $V = b h$ to find volumes of right rectangular prisms with fractional	of right rectangular prisms and right rectangular	8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles,
edge lengths in the context of solving real-world	pyramids.	about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion
and mathematical problems.	Solve real-life and mathematical problems involving angle	for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of
6.G.3. Draw polygons in the coordinate plane given	measure, area, surface area, and volume.	the three angles appears to form a line, and give an argument in terms of transversals why this is
coordinates for the vertices; use coordinates to	7.G.4. Know the formulas for the area and circumference of a	so.
find the length of a side joining points with the	circle and use them to solve problems; give an	Understand and apply the Pythagorean Theorem.
same first coordinate or the same second	informal derivation of the relationship between the	8.G.6. Explain a proof of the Pythagorean Theorem and its converse.
coordinate. Apply these techniques in the context	circumference and area of a circle.	8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world
of solving real-world and mathematical problems.	7.G.5. Use facts about supplementary, complementary,	and mathematical problems in two and three dimensions.
6.G.4. Represent three-dimensional figures using nets	vertical, and adjacent angles in a multi-step problem to	8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
made up of rectangles and triangles, and use the	write and solve simple equations for an unknown angle	
nets to find the surface area of these figures. Apply	in a figure.	Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
these techniques in the context of solving real-	7.G.6. Solve real-world and mathematical problems involving	8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-
world and mathematical problems.	area, volume and surface area of two- and three-	world and mathematical problems.
	dimensional objects composed of triangles,	
	quadrilaterals, polygons, cubes, and right prisms.	

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	Statistics and Probability		
Grade 6	Grade 7	Grade 8	
 Develop understanding of statistical variability. 6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am !?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages. 6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. 6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. Summarize and describe distributions. 6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. 6.SP.5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the nature of the attribute under investigation, including how it was measured and its units of measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall 		Grade 8Investigate patterns of association in bivariatedata.8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association.8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.8.SP.4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they	

